Cryptanalysis of RIPEMD-128

Thomas Peyrin

joint work with Franck Landelle

NTU - Singapore

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NANYANG TECHNOLOGICAL UNIVERSITY

Motivations to study RIPEMD−128

- MDx-like hash function is a very frequent design :
  - 1990' MDx (MD4, MD5, SHA−1, HAVAL, RIPEMD)
  - 2002 SHA−2 (SHA−224, . . . , SHA−512)
- Some old hash functions are still unbroken :
  - Broken MD4, MD5, RIPEMD−0
  - Broken HAVAL
  - Broken SHA−1
  - Unbroken RIPEMD−128, RIPEMD−160
  - Unbroken SHA−2
- RIPEMD−128
  - Design 15 years old.
  - unbroken 9 years after Wang's attacks [WLF+05].

General design and security notions

- A hash function $\mathcal{H}$ is often defined by repeated applications of a compression function h.
- A collision on the hash function $\mathcal{H}$ always comes from a collision on the compression function h:

$$\mathcal{H}(M) = \mathcal{H}(M^*) \Longrightarrow h(cv, m) = h(cv^*, m^*)$$

The conditions on cv and m give different kind of attacks :

Collision $cv = cv^*$ fixed and $m \neq m^*$ free.

Semi-free-start Collision $cv = cv^*$ and $m \neq m^*$ are free.

Free-start Collision $(cv, m) \neq (cv^*, m^*)$ are free.

The cryptanalysis history of MD5 is a good example of why **(semi)-free-start collisions are a serious warning.**

Results on RIPEMD-128 compression function

RIPEMD-128 parameters :

- **Digest** 128 bits
- **Steps** 64 steps (4 rounds of 16 steps each)

Known and new results on RIPEMD-128 compression function:

Target	#Steps	Complexity	Ref.
collision	48	2^{40}	[MNS12]
collision	**60**	$2^{57.57}$	**new**
collision	**63**	$2^{59.91}$	**new**
collision	**Full**	$2^{61.57}$	**new**
non-randomness	52	2^{107}	[SW12]
non-randomness	**Full**	$2^{59.57}$	**new**

In this talk

Function RIPEMD-128 compression function

Attack a semi-free-start collision

Find $cv, m \neq m^* \,/\, h(cv, m) = h(cv, m^*)$.

Strategy

- Choose a message difference $\delta_m = m \oplus m^*$
  $\rightarrow$ new message difference used
- Find a differential path on all intermediate state variables
  $\rightarrow$ new type of differential path with two non-linear parts
- Find conforming cv and m
  $\rightarrow$ new branch merging technique for collision search

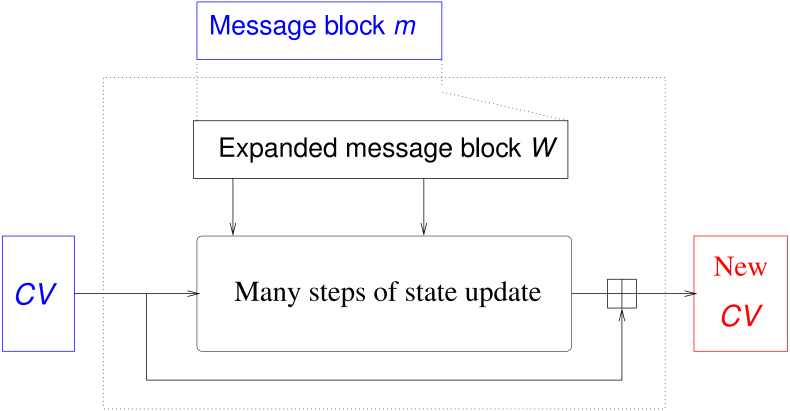
Outline

1. Description of RIPEMD-128
2. Finding a differential path
   - Finding a message difference
   - Finding the non-linear part
3. Finding a conforming pair
   - Generating a starting point
   - Merging the 2 branches
4. Conclusion

Outline

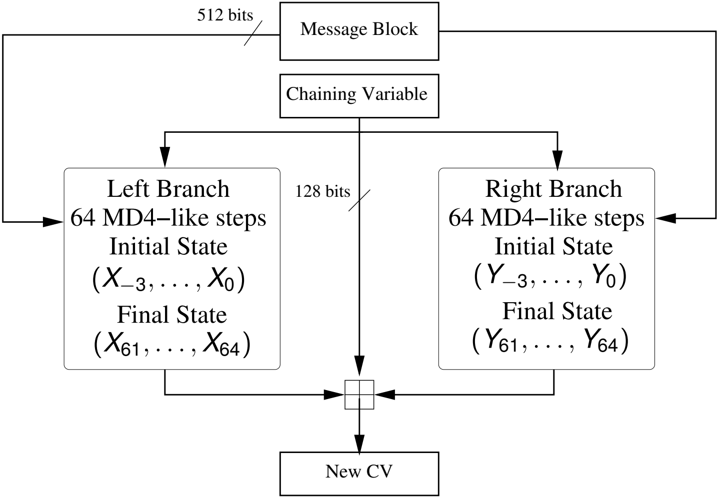
A compression function

$$m = m_0 || m_1 || \cdots || m_{15}$$



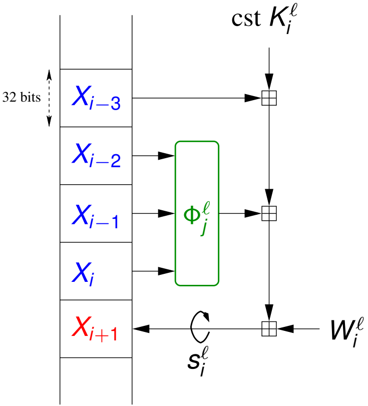
Compression Function

Overview of RIPEMD−128 compression function



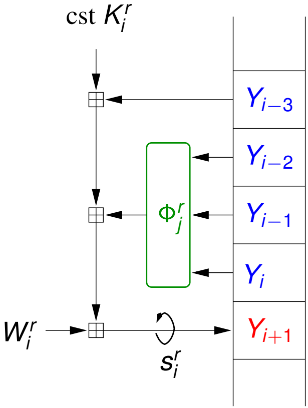
The step function

$$W_i^r = m_{\pi_j^r(i)}$$



Left Branch - step i, round j

$$W_i^\ell = m_{\pi_j^\ell(i)}$$



Right Branch - step i, round j

The boolean functions

Boolean functions in RIPEMD-128:

- $\mathrm{XOR}(x, y, z) := x \oplus y \oplus z$,
- $\mathrm{IF}(x, y, z) := x \wedge y \oplus \bar{x} \wedge z$
- $\mathrm{ONX}(x, y, z) := (x \vee \bar{y}) \oplus z$

Steps i	Round j	$\Phi_j^{\ell}(x, y, z)$	$\Phi_j^{r}(x, y, z)$
0 to 15	0	$\mathtt{XOR}(x, y, z)$	$\mathtt{IF}(z, x, y)$
16 to 31	1	$\mathtt{IF}(x, y, z)$	$\mathtt{ONX}(x, y, z)$
32 to 47	2	$\mathtt{ONX}(x, y, z)$	$\mathtt{IF}(x, y, z)$
48 to 63	3	$\mathtt{IF}(z, x, y)$	$\mathtt{XOR}(x, y, z)$

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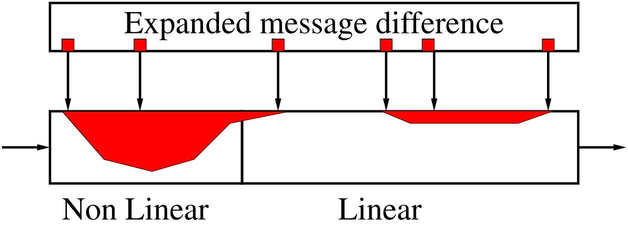
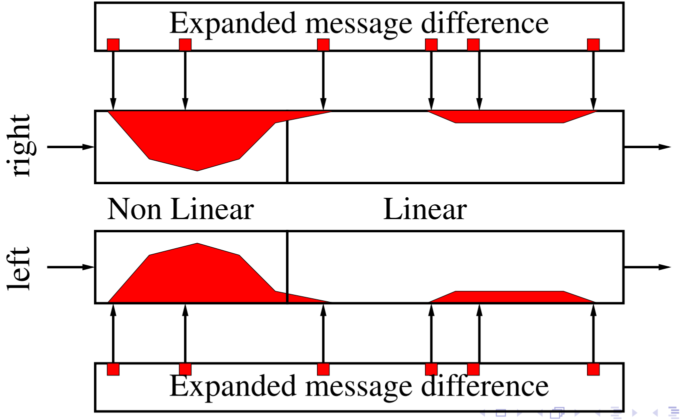
The classical strategy (example SHA-1)

1. Find a message difference δ_m and a differential path with high probability on the middle and last steps (ideally after the first round).
2. Find a “realistic” non-linear differential path on the first steps (ideally on the first round for a semi-free-start collision).
3. Find a chaining variable *cv* and a message *m* such that the state differential path is followed (use special freedom degrees tricks like neutral bits, message modification, boomerangs, etc.).



The classical strategy (example RIPEMD-128)

1. Find a message difference δ_m and a differential path with high probability on the middle and last steps for both branches.
2. Find a "realistic" non-linear differential path on the first steps.
3. Find a conforming chaining variable *cv* and a message *m*.



What shape should have the differential path ?

Boolean functions can help to control the diff. propagation.

Properties of the boolean functions:

- XOR : no control of differential propagation
- ONX: some control of differential propagation and permits low diffusion.
- IF : a good control of differential propagation and permits **no** diffusion.

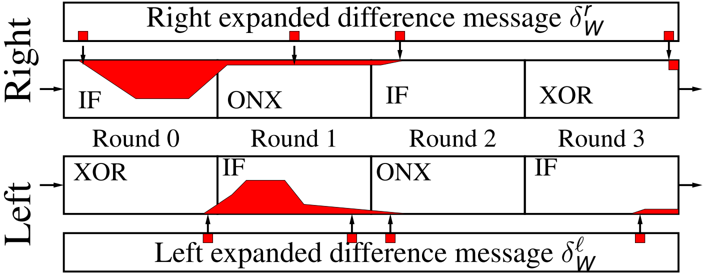
Steps i	Round j	$\Phi^l_j(x,y,z)$	$\Phi^r_j(x,y,z)$
0 to 15	0	$\texttt{XOR}(x,y,z)$	$\texttt{IF}(z,x,y)$
16 to 31	1	$\texttt{IF}(x,y,z)$	$\texttt{ONX}(x,y,z)$
32 to 47	2	$\texttt{ONX}(x,y,z)$	$\texttt{IF}(x,y,z)$
48 to 63	3	$\texttt{IF}(z,x,y)$	$\texttt{XOR}(x,y,z)$

Outline

Choosing the message block difference

Goals keep low ham. weight on the expanded message block

Choice Put a difference on a single word of message



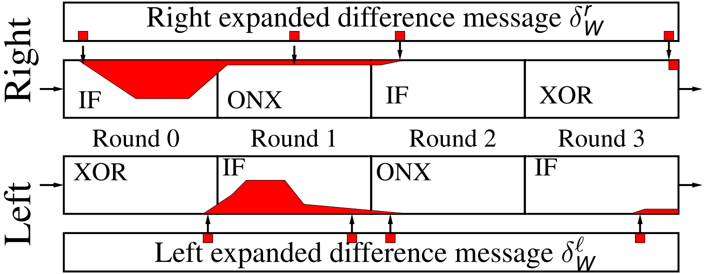
With the message block difference on m_{14}:

- “no difference” on rounds with XOR function.
- Non-linear differential paths are in the round with IF

Choosing the message block difference

m_{14} is really “**magic**” with regards to our criteria.

However, **how to handle these** **two** **non-linear parts which are in different branches, and not in the first round** ?



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Automatic tool on generalized conditions

We implemented a tool similar to [CR06] for SHA-1 that uses generalized conditions.

Hexa	(b, b^*) Notation	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
0xF	?	✓	✓	✓	✓
0x9	-	✓			✓
0x6	x		✓	✓	
0x1	0	✓			
0x2	u		✓		
0x4	n			✓	
0x8	1				✓

Where

- b: a bit during the treatment the message m
- b^*: the same bit for the second message m^*.

Left branch

```
Step                Xi                                  Wi                         Πi
13: -------------------------------- | --------------------------------  13
14: -------------------------------- | x-------------------------------  14
15: ???????????????????????????????? | --------------------------------  15
16: ???????????????????????????????? | --------------------------------   7
17: ???????????????????????????????? | --------------------------------   4
18: ???????????????????????????????? | --------------------------------  13
19: ???????????????????????????????? | --------------------------------   1
20: ???????????????????????????????? | --------------------------------  10
21: ???????????????????????????????? | --------------------------------   6
22: ???????????????????????????????? | --------------------------------  15
23: ???????????????????????????????? | --------------------------------   3
24: ???????????????????????????????? | --------------------------------  12
25: ???????????????????????????????? | --------------------------------   0
26: -------u------------------------ | --------------------------------   9
27: 1------0-----u------------------ | --------------------------------   5
28: 0------1-----0------------------ | --------------------------------   2
29: n------------1------------------ | x-------------------------------  14
30: u------------------------------- | --------------------------------  11
31: u------------------------------- | --------------------------------   8
32: 1------------------------------- | --------------------------------   3
33: -------------------------------- | --------------------------------  10
34: -------------------------------- | x-------------------------------  14
35: -------------------------------- | --------------------------------   4
```

Left branch

```
Step            Xi                                      Wi                    Πi
13: -------------------------------- | -------------------------------- 13
14: -------------------------------- | x------------------------------- 14
15: -----------------------n-------- | -------------------------------- 15
16: -----------unnnn-------0-------- | --------------------------------  7
17: -------n---00000-------1-------- | --1-----------------------------  4
18: -------0---01111---------------- | -------------------------------- 13
19: ---u---1-------n------------1--- | --------------------------------  1
20: ---0-----------0------------0--- | -------------------------------- 10
21: ---1-----------1------------n--- | --------------------------------  6
22: ---------------unnnn--------0--- | -------------------------------- 15
23: ---------------00000--------u--- | --------------------------------  3
24: -------------n-11101--------1--- | -------------------------------- 12
25: -----------n-0--------------1--- | --------------------------------  0
26: -------u---0-1------------------ | --------------------------------  9
27: 1------0---1-u------------------ | --------------------------------  5
28: 0------1-----0------------------ | --------------------------------  2
29: n------------1------------------ | x------------------------------- 14
30: u------------------------------- | -------------------------------- 11
31: u------------------------------- | --------------------------------  8
32: 1------------------------------- | --------------------------------  3
33: -------------------------------- | -------------------------------- 10
34: -------------------------------- | x------------------------------- 14
35: -------------------------------- | --1-----------------------------  4
```

Right branch

```
Step              Yi                                           Wi                              πi
   : --------------------------------
   : --------------------------------
   : --------------------------------
   : -------------------------------- | --------------------------------  5
01: -------------------------------- | x------------------------------- 14
02: ???????????????????????????????? | --------------------------------  7
03: ???????????????????????????????? | --------------------------------  0
04: ???????????????????????????????? | --------------------------------  9
05: ???????????????????????????????? | --------------------------------  2
06: ???????????????????????????????? | -------------------------------- 11
07: ???????????????????????????????? | --------------------------------  4
08: ???????????????????????????????? | -------------------------------- 13
09: ???????????????????????????????? | --------------------------------  6
10: ???????????????????????????????? | -------------------------------- 15
11: ???????????????????????????????? | --------------------------------  8
12: ???????????????????????????????? | --------------------------------  1
13: ???????????????????????????????? | -------------------------------- 10
14: ???????????????????????????????? | --------------------------------  3
15: -------u------------------------ | -------------------------------- 12
16: -------u----u------------------- | --------------------------------  6
17: -----u-0----u------------------- | -------------------------------- 11
18: -----u------0------------------- | --------------------------------  3
19: 0----0-------------------------- | --------------------------------  7
20: u------------------------------- | --------------------------------  0
```

Right branch

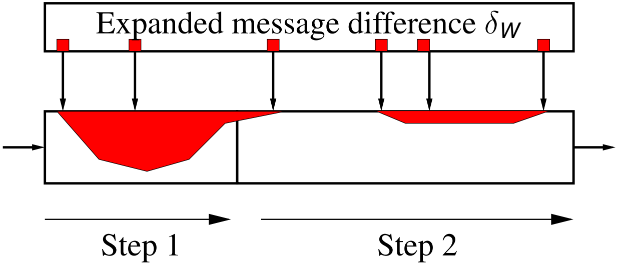
```
Step              Yi                                         Wi                    πi
   : --------------------------------
   : --------------------------------
   : --------------------------------
   : -----------------------0-------- | --------------------------------  5
01:  -----------------------1-------- | x-------------------------------  14
02:  -----------------------n-------- | --------------------------------  7
03:  -------------------------------- | --------------------------------  0
04:  --0000000----------------------- | --------------------------------  9
05:  --1111111----------------------- | --------------------------------  2
06:  --nuuuuuu----------------------- | --------------------------------  11
07:  --01-----------------------0-000 | --1-----------------------------  4
08:  -01------------------------0-011 | --------------------------------  13
09:  -1----------------10-0-----n-nnn | --------------------------------  6
10:  1n010000----------11-1---------- | --------------------------------  15
11:  00111111-----00--0nu-n---------- | --------------------------------  8
12:  nuuuuuuu-----11--11--0---------- | --------------------------------  1
13:  -------1-----nn--un--u---------- | --------------------------------  10
14:  -------1----01----u------------- | --------------------------------  3
15:  -------u----10----0------------- | --------------------------------  12
16:  -----0-u----u------------------- | --------------------------------  6
17:  -----u-0----u------------------- | --------------------------------  11
18:  -----u------0------------------- | --------------------------------  3
19:  0----0-------------------------- | --------------------------------  7
20:  u------------------------------- | --------------------------------  0
```

Outline

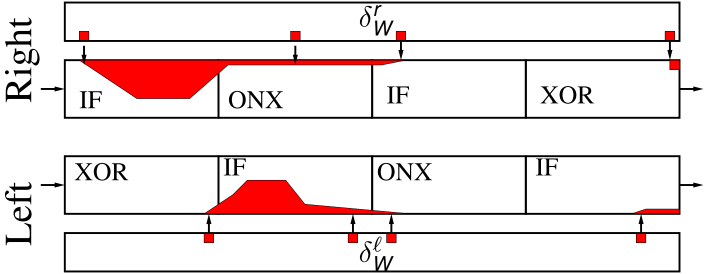
Following a classical differential path

A classical collision search is composed of two subparts:

- **step 1** handling the low-probability non-linear parts using the message block freedom
- **step 2** the remaining steps in both branches are verified probabilistically



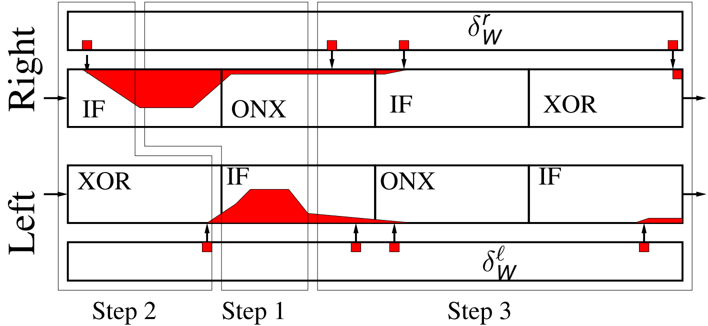
Finding a conforming pair



Our collision search is composed of three subparts:

- step 1 Satisfy the two non-linear parts using the freedom from both branches internal states and a few message words
- step 2 From this **starting point**, merge the two branches using some remaining free message words
- step 3 Handle probabilistically the linear part in both branches

Finding a conforming pair



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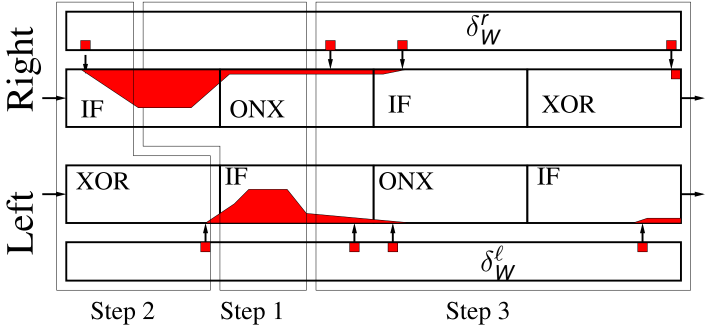
Satisfying the two non-linear parts simultaneously (step 1)



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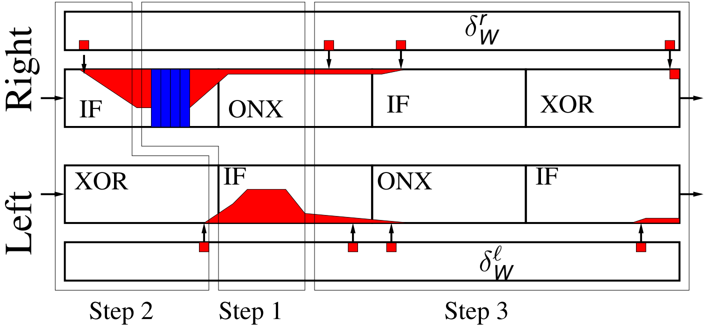

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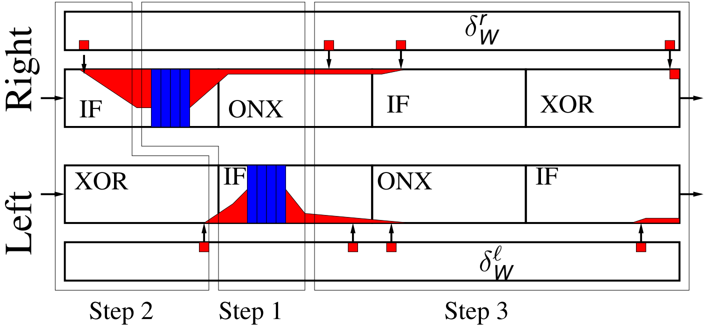

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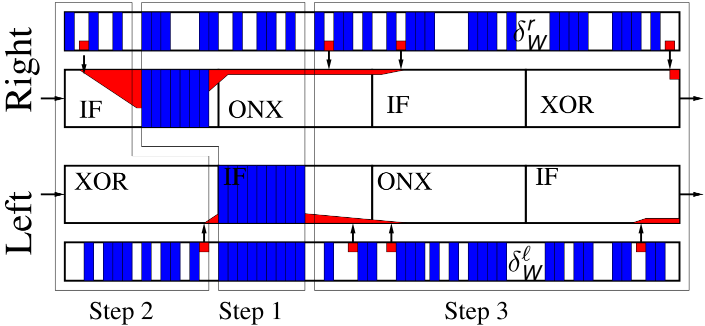

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Handling probabilistically the linear parts (step 3)

Probabilities of the linear parts are fixed after the first step:

- The probability of the left branch is 2^{-15}.
- The probability of the right branch is $2^{-14.32}$.
- one extra bit condition in order to get a collision when adding the two branches
- $\rightarrow$ The overall probability for collision is $2^{-30.32}$.

(these probabilities have been verified experimentally)

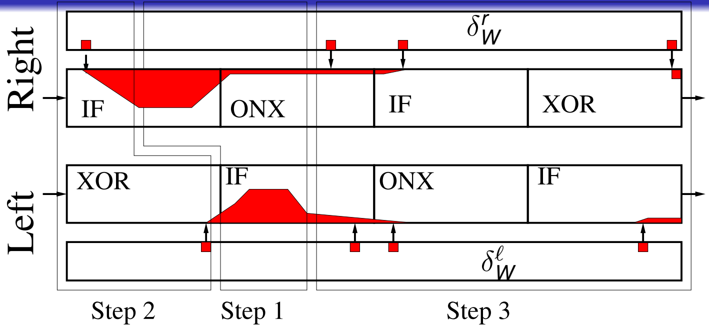
Our collision search is composed of three subparts:

step 1 Satisfy the two non-linear parts using the freedom from both branches internal states and a few message words

step 2 From this **starting point**, merge the two branches using some remaining free message words

step 3 Handle probabilistically the linear part in both branches

Handling probabilistically the linear parts (step 3)



→ we need to obtain $2^{30.32}$ solutions of the merging system

Our collision search is composed of three subparts:

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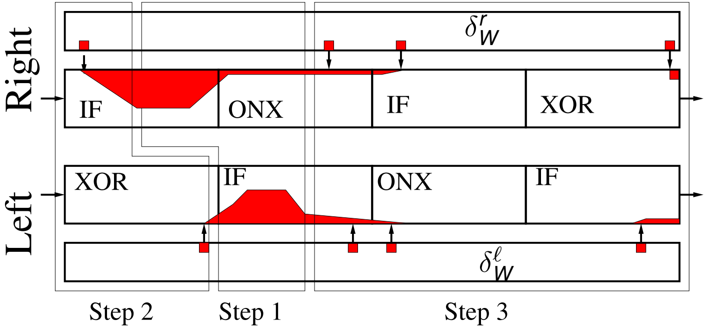
Outline

Merging the two branches (step 2)



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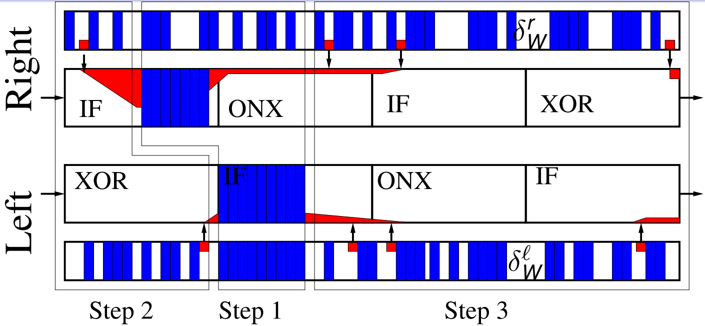
The starting point



What is fixed ?

Message $m_{12}, m_3, m_{10}, m_1, m_8, m_{15}, m_6, m_{13}, m_4, m_{11}, m_7$.

Left State $(X_{12}, \ldots, X_{24})$

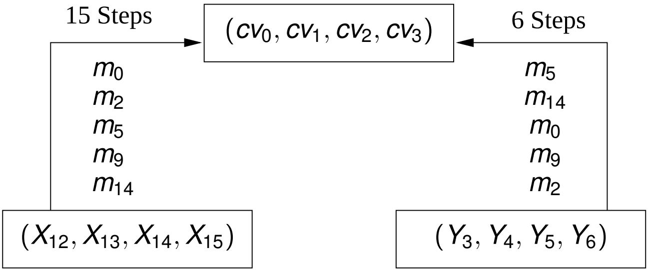
Right State $(Y_3, Y_4, \ldots, Y_{14})$.

What is free ?

Message $m_0, m_2, m_5, m_9, m_{14}$.

Prepare the merging system

The system is quite complex:



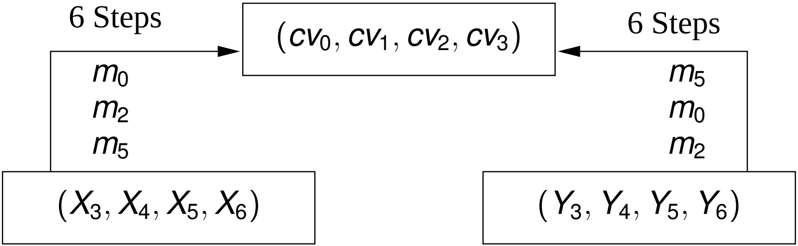
The probability that a random choice of $m_0, m_2, m_5, m_9, m_{14}$ gives a solution is

$$2^{-128}$$

Reducing the merging system

- in the search for a starting point (step 1), we chose m_{11} such that: $Y_3 = Y_4$
- randomly chose a m_{14} value and deduce m_9 such that: $X_5^{\ggg 5} \boxminus m_4 = \texttt{0xffffffff}$

$\rightarrow$ **the system becomes much simpler and represents less steps of the compression function.**



Solving the merging system

The goal now is to find m_0, m_2, m_5 such that

$$X_i = Y_i \text{ for } i \in \{-3, -2, -1, 0\}$$

	X_0	Y_0	X_{-1}	Y_{-1}	X_{-2}	Y_{-2}	X_{-3}	Y_{-3}
m_2		✓	✓	✓	✓	✓	✓	✓
m_0		✓					✓	
m_5					✓		✓	✓

To solve the merging system:

1. find a value of m_2 that verifies $X_{-1} = Y_{-1}$
2. deduce m_0 to fulfill $X_0 = Y_0$
3. obtain m_5 to satisfy a combination of $X_{-2} = Y_{-2}$ and $X_{-3} = Y_{-3}$
4. finally the 4^{th} equation is verified with probability 2^{-32}

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m_0		✓					✓	
m_5					✓		✓	✓

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m_0		✓					✓	
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m_0		✓					✓	
m_5					✓		✓	✓

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	X_0	Y_0	X_{-1}	Y_{-1}	X_{-2}	Y_{-2}	X_{-3}	Y_{-3}
m_2		✓	✓	✓	✓	✓	✓	✓
m_0		✓					✓	
m_5					✓		✓	✓

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4. finally the 4^{th} equation is verified with probability 2^{-32}

Complexity of the semi-free-start collision attack

- Solving the merging system costs 19 RIPEMD-128 step computations ($19/128$ of the compression function cost).
- The probability of success of the merging is 2^{-34} (because of 4^{th} equation and 2 extra hidden bit conditions)
- We need to find $2^{30.32}$ solutions of the merging system.

The **total complexity** is therefore

$$19/128 \times 2^{34} \times 2^{30.32} \simeq 2^{61.57}$$

calls to the compression function.

Outline

Conclusion

This work:

- a new cryptanalysis technique for parallel branches based functions
- a collision attack on the full compression function of RIPEMD−128
- a distinguisher on the hash function of RIPEMD−128
- a LOT of details (many not described here)

Perspectives:

- improvements of this technique
- an example of collision for RIPEMD−128?
- apply to other 2-branch hash functions
- what about RIPEMD−160?

Cryptanalysis of RIPEMD-160

Thomas Peyrin

joint work with F. Mendel, M. Schläffer, L. Wang and S. Wu

(accepted at Asiacrypt 2013)

ASK 2013

Weihai, China - August 29 , 2013

NANYANG TECHNOLOGICAL UNIVERSITY

Results on RIPEMD-160 compression function

RIPEMD-160 parameters :

Digest 160 bits

Steps 80 steps (5 rounds of 16 steps each)

Known and new results on RIPEMD-160 compression function:

Target	#Steps	Complexity	Ref.
semi-free-start collision	36	low (practical)	[MNS12]
1st round			
semi-free-start collision	**36**	$2^{70.4}$	**new**
semi-free-start collision	**42**	$2^{75.5}$	**new**

RIPEMD-160 >> RIPEMD-128

Why are the improvements far less impressive for RIPEMD-160?

The technique we applied on RIPEMD-128 is much harder to apply on RIPEMD-160:

- finding non-linear parts is more difficult than for RIPEMD-128
- evaluating the probability of a differential path is hard (because two additions are interlinked)
- ... so more complicated to have a global view of what will and what won't work when trying to organize the attack

On top of that, RIPEMD-160 has

- better diffusion (impossible to force no diffusion, even in IF rounds)
- more steps ...

Thank you for your attention !

We are looking for good PhD students
in symmetric key crypto.

If interested, please contact me at:
thomas.peyrin@ntu.edu.sg

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