SHA-1 : Beating a Dead Horse

Thomas Peyrin

(joint work with Gaëtan Leurent)

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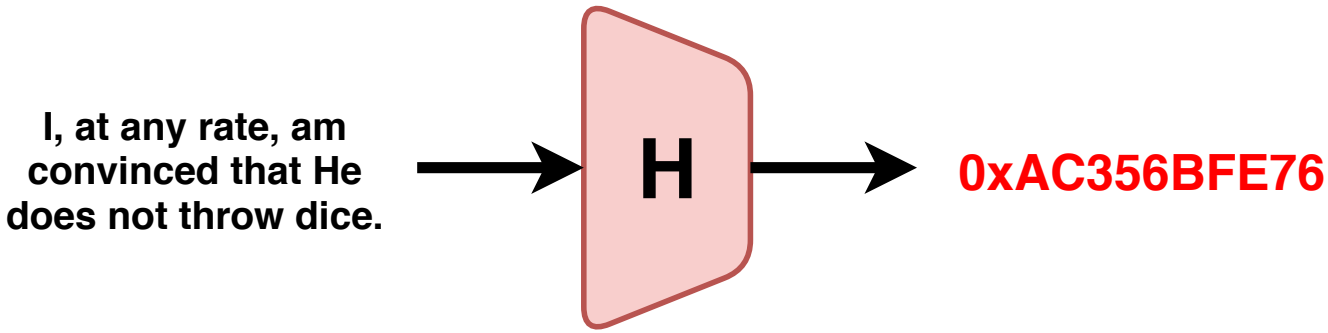
Outline

What is a Hash Function ?



- H maps an **arbitrary length input** (the message M) to a **fixed length output**. Typically $n = 128$ (MD5), $n = 160$ (SHA-1) or $n = 256$ bits (SHA-256).
- no secret parameter.
- H must be easy to compute.

The security goals

pre-image resistance :

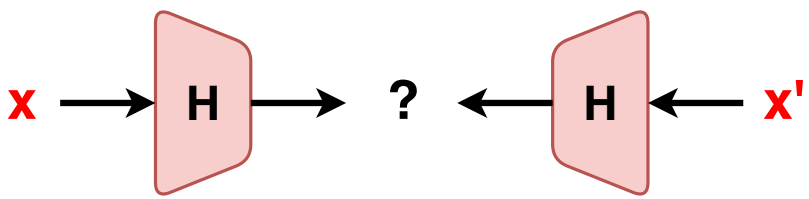
given an output challenge y, the attacker can not find a message x such that $H(x) = y$, in less than $\theta(2^n)$ operations.

2nd pre-image resistance :

given a challenge (x, y) so that $H(x) = y$, the attacker can not find a message $x' \neq x$ such that $H(x') = y$, in less than $\theta(2^n)$ operations.

collision resistance :

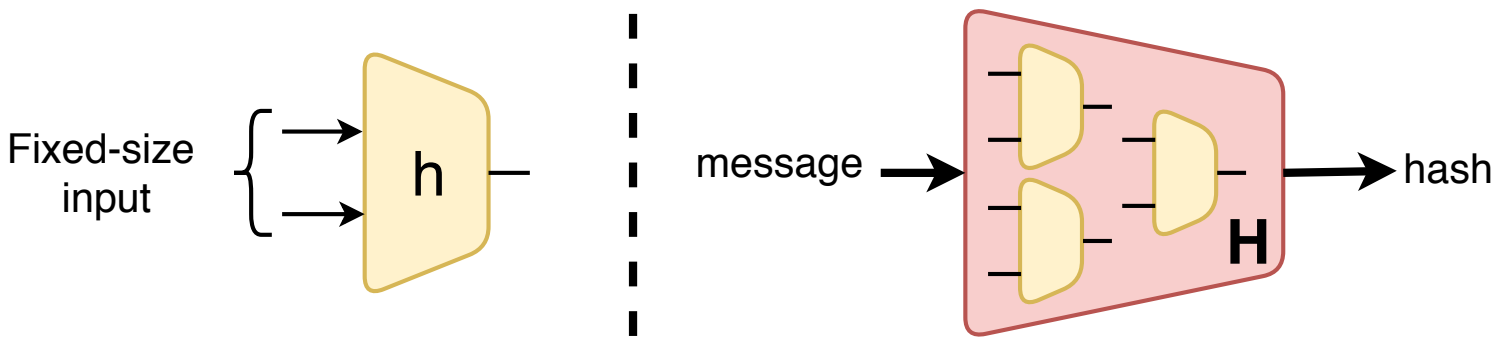
the attacker can not find two messages (x, x') such that $H(x) = H(x')$, in less than $\theta(2^{n/2})$ operations (a generic attack with the birthday paradox exists [Yuval-79]).



General hash construction

For historical reasons, most hash functions are composed of two elements :

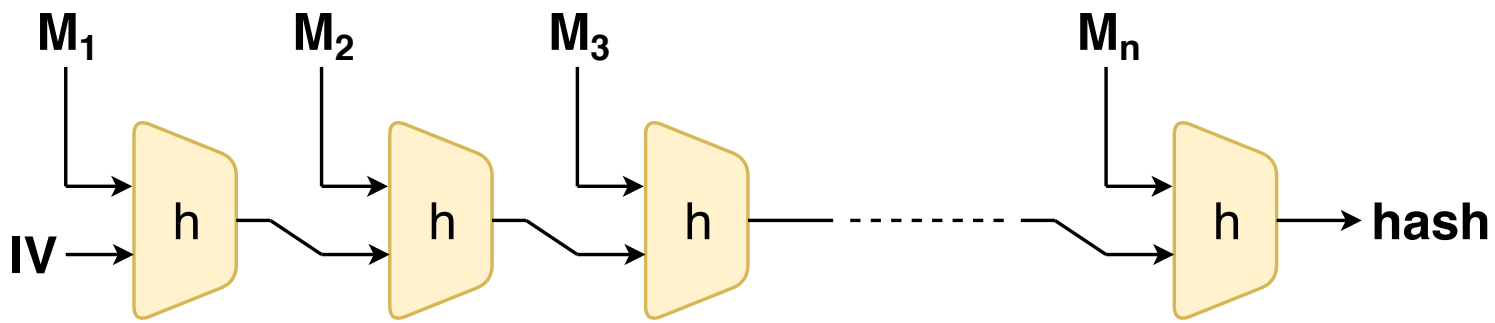
- **a compression function** h **:** a function for which the input and output size is fixed.
- **a domain extension algorithm :** an iterative process that uses the compression function h so that the hash function H can handle inputs of arbitrary length.



The Merkle-Damgård domain extension algorithm

The most famous domain extension algorithm used is called the **Merkle-Damgård** [Merkle Damgård-89] iterative algorithm.

$$\mathbf{pad(M) = M_1 \,||\, M_2 \,||\, M_3 \,||\, \ldots \,||\, M_n}$$



The compression function *h* now takes two fixed-size inputs, the incoming chaining variable *cv* and the message block *m*, and outputs a new chaining variable.

The sad story of MD5

The cryptanalysis history of MD5 is a good example of why **(semi)-free-start collisions are a serious warning**.

- 1992 MD5 RFC published
- 1993 pseudo-collision on the compression function [BB93]
- 1994 semi-free-start collision on the compression function [Dob96]
- 2004 practical collisions on the hash function [WFL04]
- 2007 chosen-prefix collisions and colliding X.509 certificates [SLW07]
- 2009 rogue CA certificates for RapidSSL [S+09] (used chosen-prefix collision)
- 2010-2012 Flame malware (used chosen-prefix collision)

What about SHA-1 ?

The current situation of SHA-1 :

1995	SHA-1 NIST FIPS 180-1 published : basically SHA-0 (1993) with a very small twist
2005	theoretical collision attack on the full hash function [WYY05] - 2^{69}
2006-2011	lots of works computing actual collisions for reduced-round versions of SHA-1
2015	free-start collision on the full compression computed [SKP15] - 2^{57}
2017	full collision on the full hash function computed [SBKAM17] - $2^{64.7}$
2019	chosen-prefix collision attack [LP19] - $2^{67.2}$
New	chosen-prefix collision attack - $2^{63.7}$ + PGP/GnuPG key-certification forgery

Motivations to study SHA-1

Why still studying SHA-1 ?

Design from NSA, **Standard** from NIST, **Still used worldwide** despite deprecation efforts (major browsers now refusing to connect to servers still using SHA-1- based certificates) :

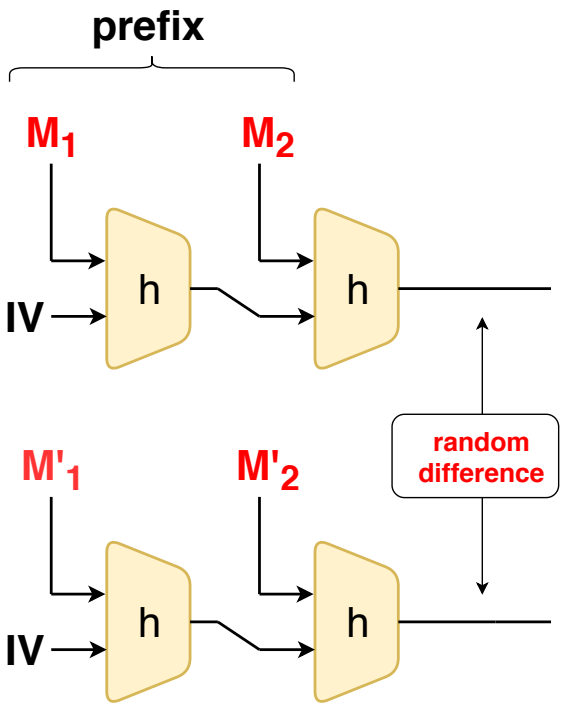
- more than 5% of Alexa's top 1 million **prefer** to use SHA-1 to authenticate TLS handshake messages (including www.skype.com)
- about 30000 servers with SHA-1 certificates (out of 720000 servers with HTTPS support)
- other protocols : about 1 million out of 4.55 millions mail servers (with IMAPS) use a SHA-1 certificate
- it is still possible to buy a SHA-1 certificate from a trusted root, and some can be found in the wild
- the "Mail" application included in Windows 10 still accepts SHA-1 certificates without warnings when opening an IMAPS connection

Yet another push is perhaps needed to accelerate the retirement of SHA-1

What are chosen-prefix collisions ?

Chosen-prefix collision attack

The attacker is first challenged with **two message prefixes** P and P', and its goal is to compute two messages M and M' to create the **collision** $H(P||M) = H(P'||M')$, where $||$ denotes concatenation



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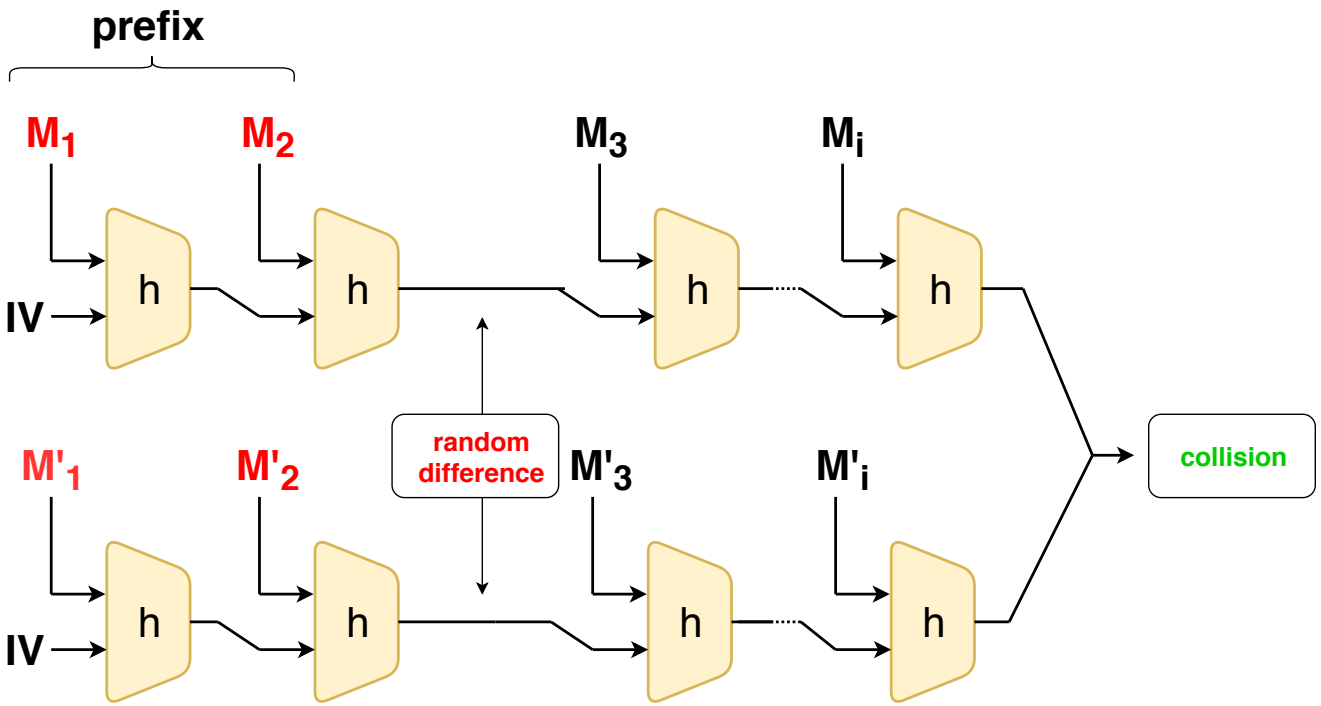

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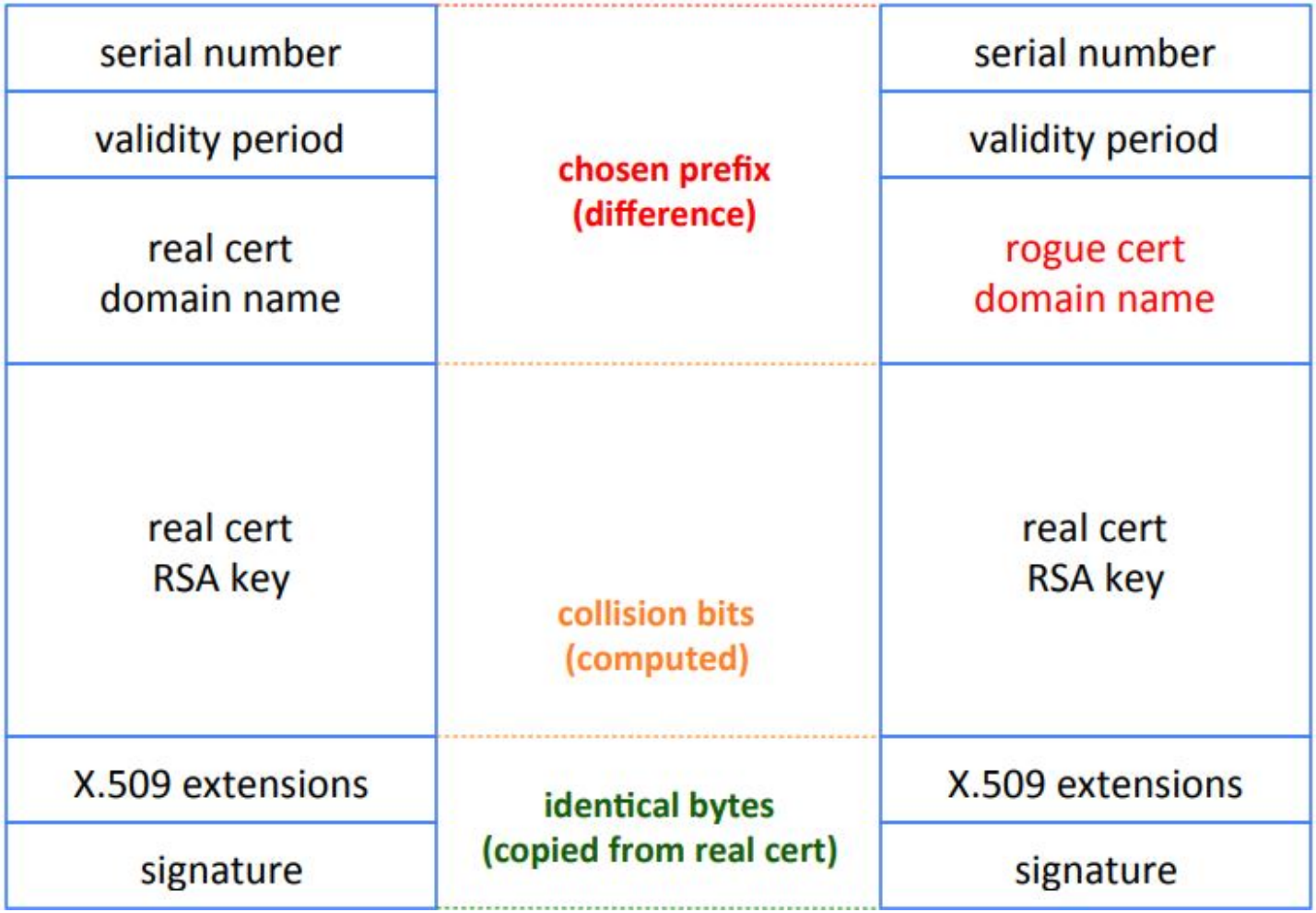
- **much more powerful** than a simple collision attack (i.e. rogue CA certificate)
- supposedly much harder than a simple collision attack (currently for MD5, from 2^{16} to 2^{39})
- birthday attack can apply, thus generic cost remains $2^{n/2}$ (i.e. 2^{80} in the case of SHA-1)

Why chosen-prefix collisions ?

Colliding SSL certificates [S+09] :



Alex Sotirov https://trailofbits.files.wordpress.com/2012/06/flame-md5.pdf

Chosen-prefix collisions for SHA−1

Previous status of chosen-prefix collisions on SHA−1 :

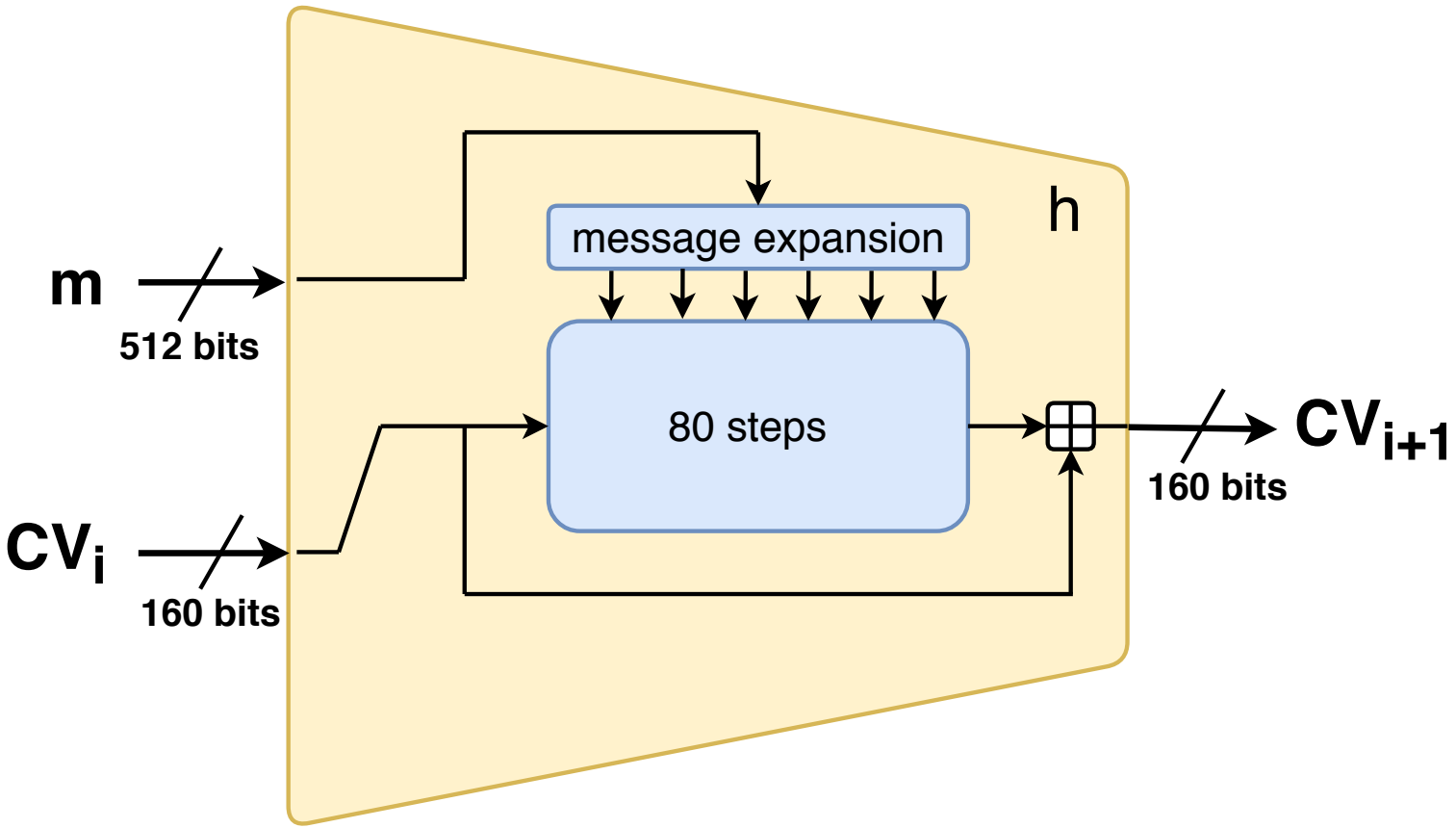
- **previous best known chosen-prefix attack against SHA−1 required $2^{77.1}$ computations** [S13]
  (a factor 8 better than generic attack).
- ... while classical collision could be found with $2^{64.7}$ computations.
- one can't apply directly the SHA−1 collision attack, because of the random state difference due to the challenge prefix.

Can we reduce the gap and make chosen-prefix collisions practical for SHA−1 ?

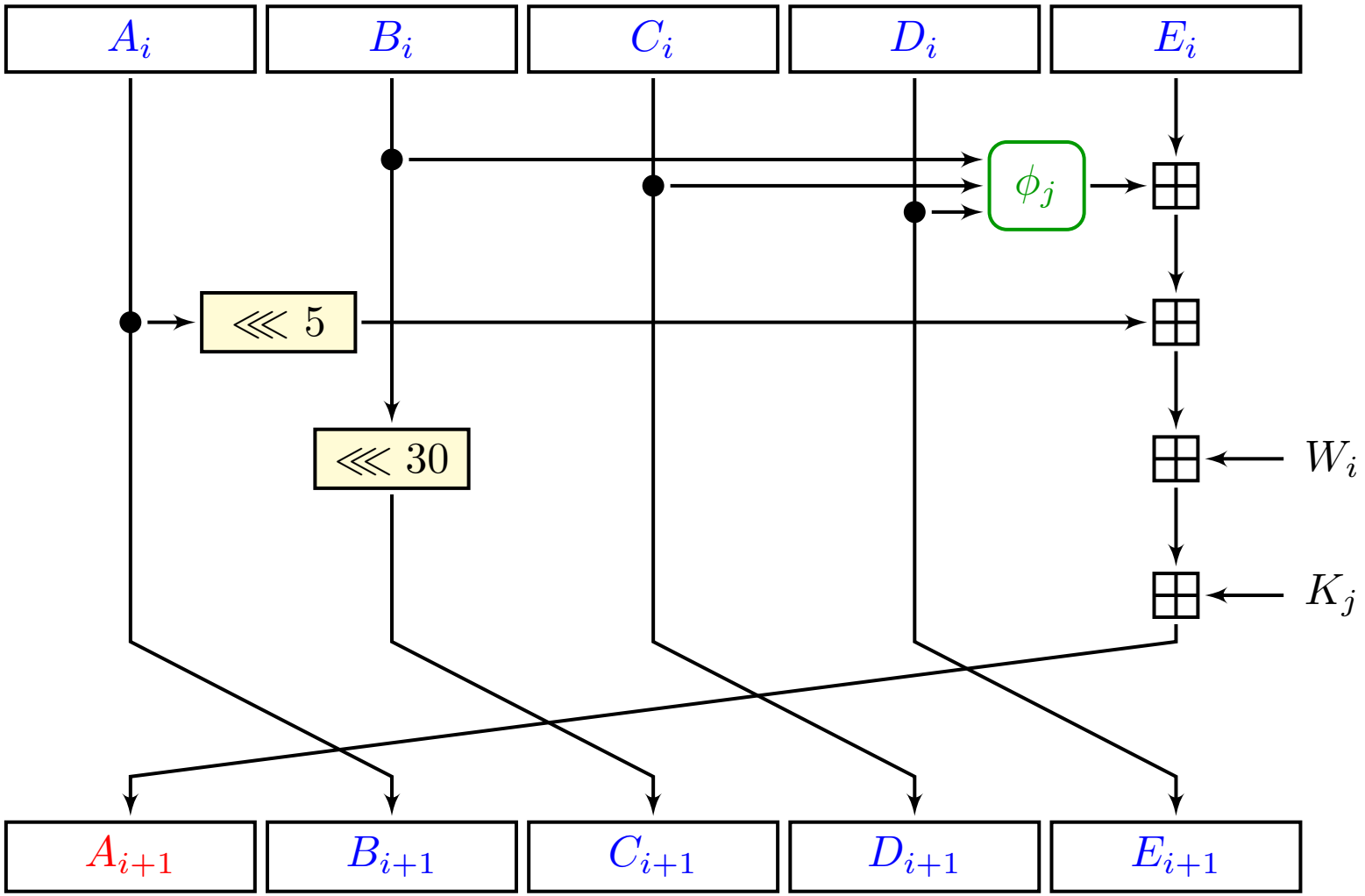
Outline

The SHA-1 compression function

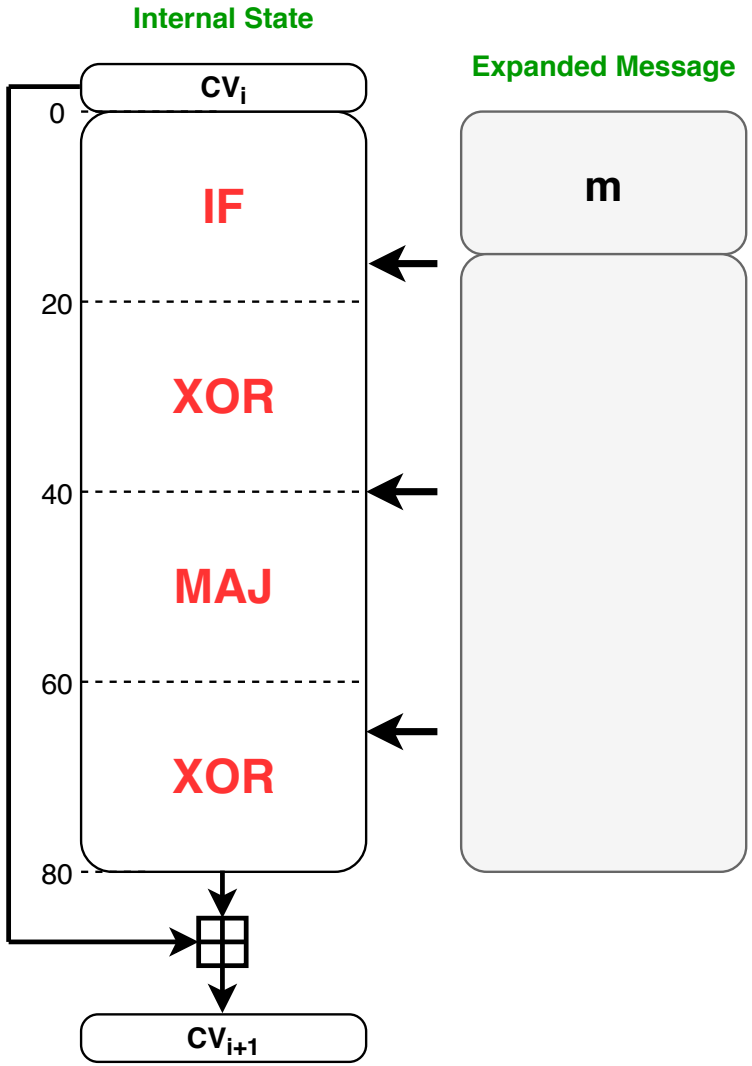
$$m = M_0 || M_1 || \cdots || M_{15}$$

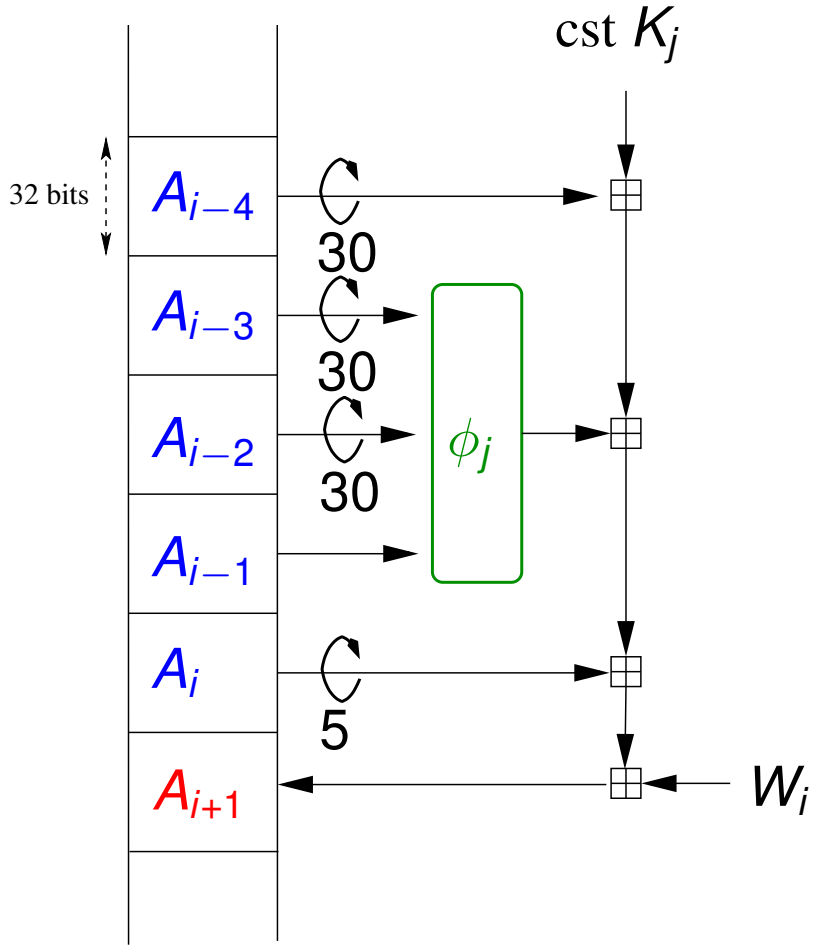


The SHA-1 step function



The SHA−1 step function : alternative representation



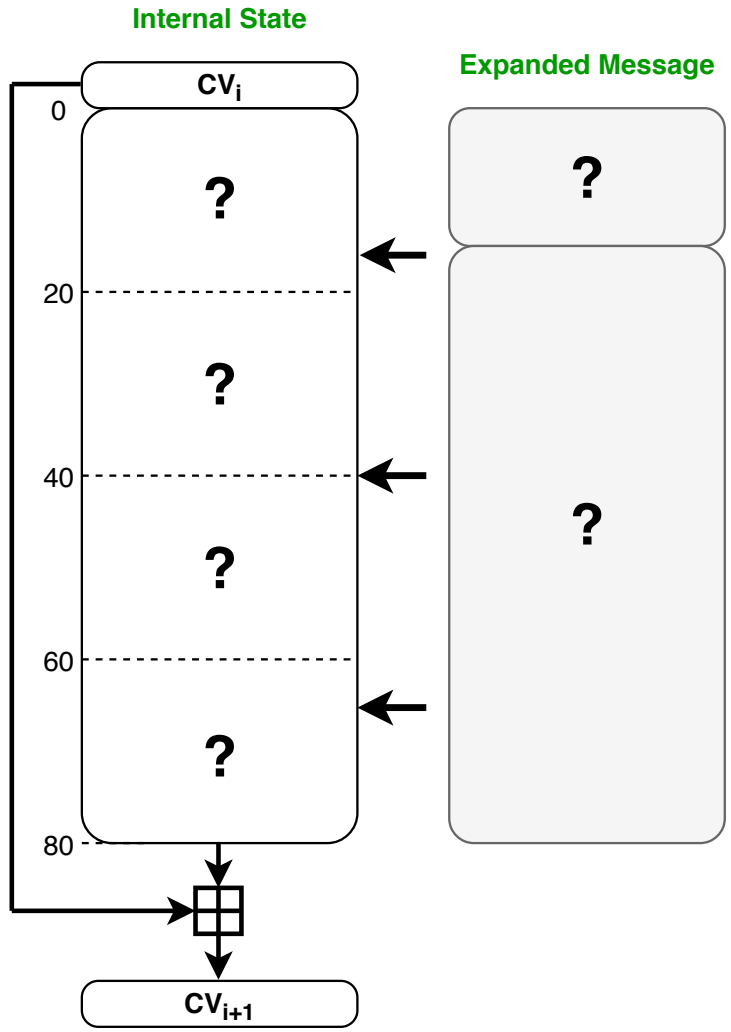


step i, round j

Outline

Goal : Find a collision for SHA-1

- **Find a linear path for rounds 16-80, using local collisions** this will fix the entire message difference, and the internal state difference for steps 16-80
- **Find a non-linear path for steps 1-15, using heuristic algorithm** this will fix the internal state difference for rounds 1-15
- **Prepare the collision search speed-up techniques** by using the freedom degrees available
- **Launch the collision search !**



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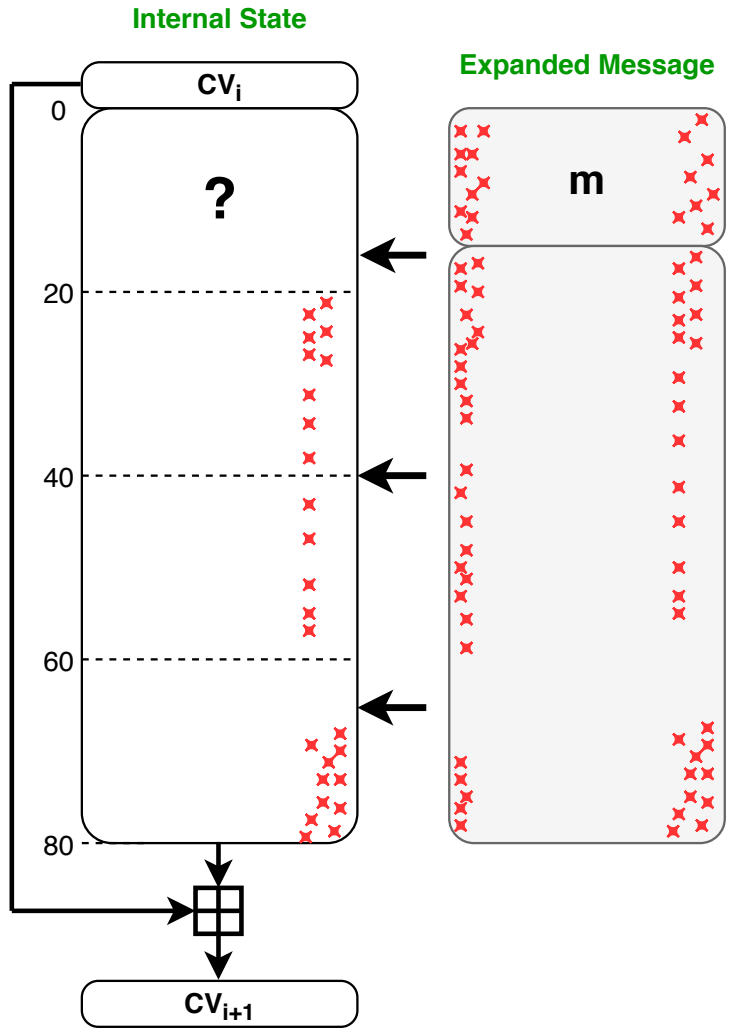

Structure of the differential path

Why did we remove the 16 first steps ?

- it allows to **avoid impossibilities due to the IF function**
- impossible to get a one-block collision with good probability : necessary to use several blocks
- A non-linear part allows to **start from any incoming difference in the chaining variable** : 2-block collision

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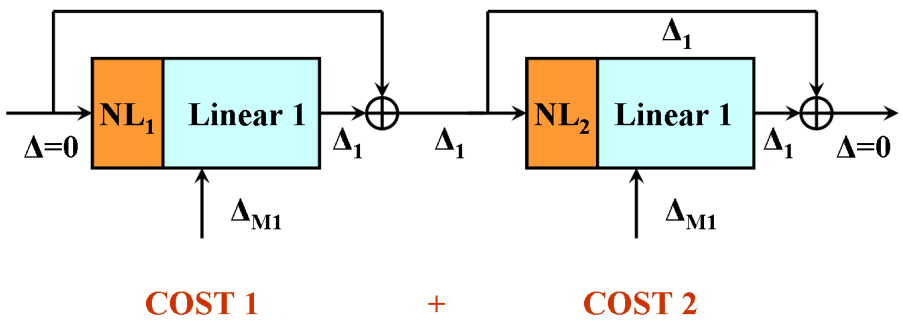

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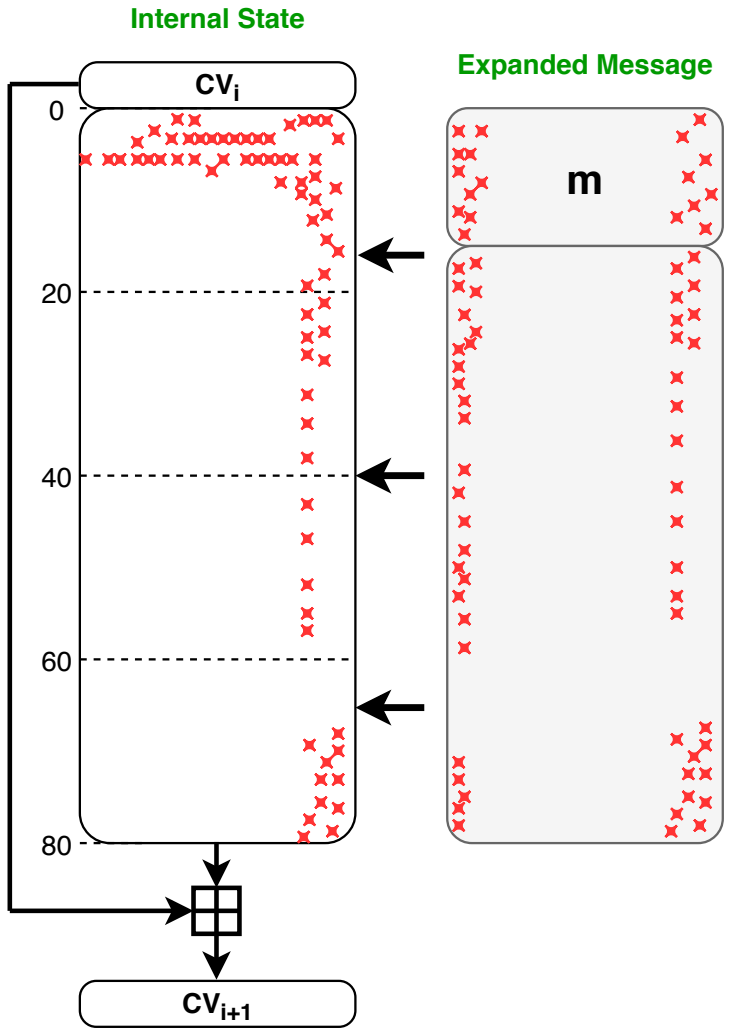
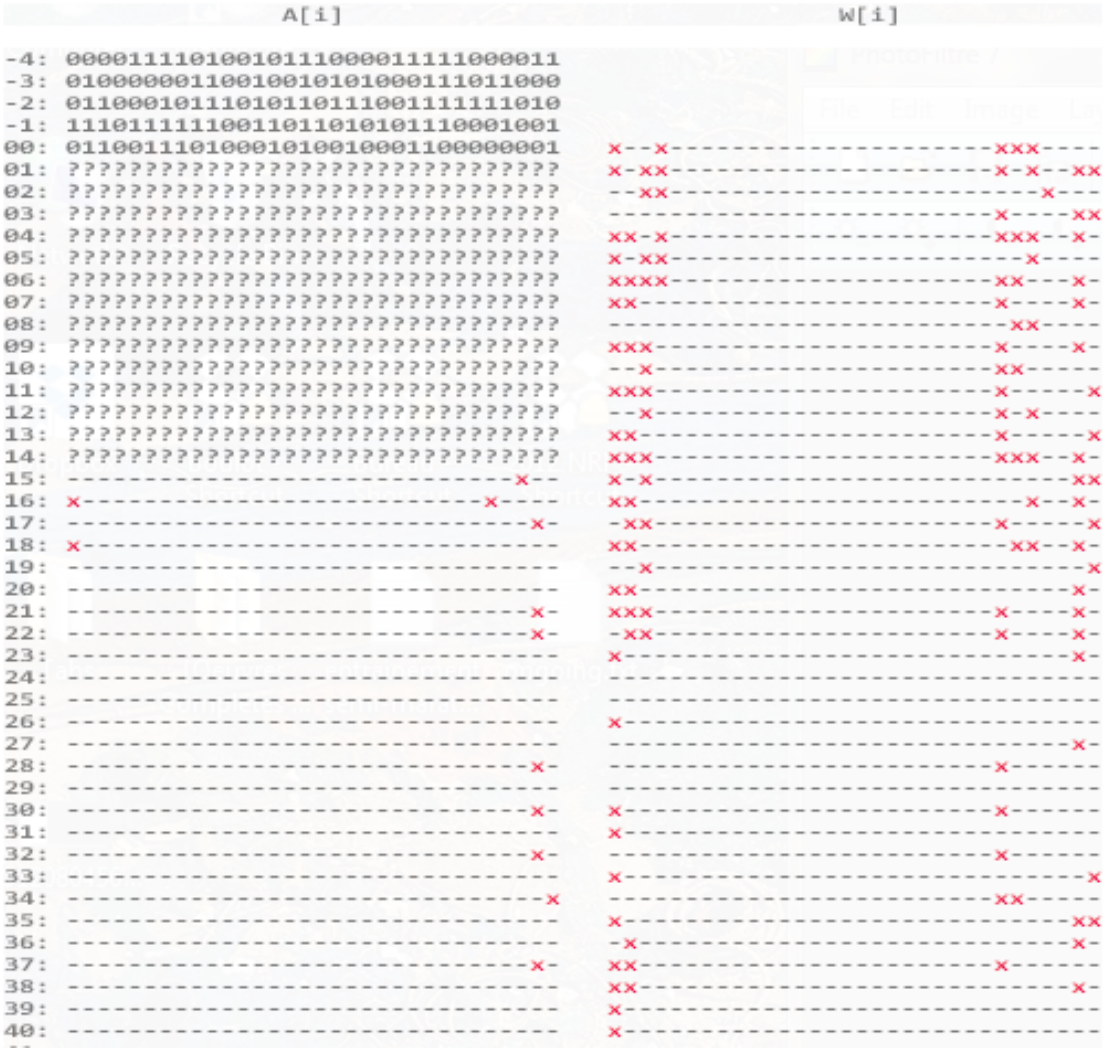


Example of a non-linear path



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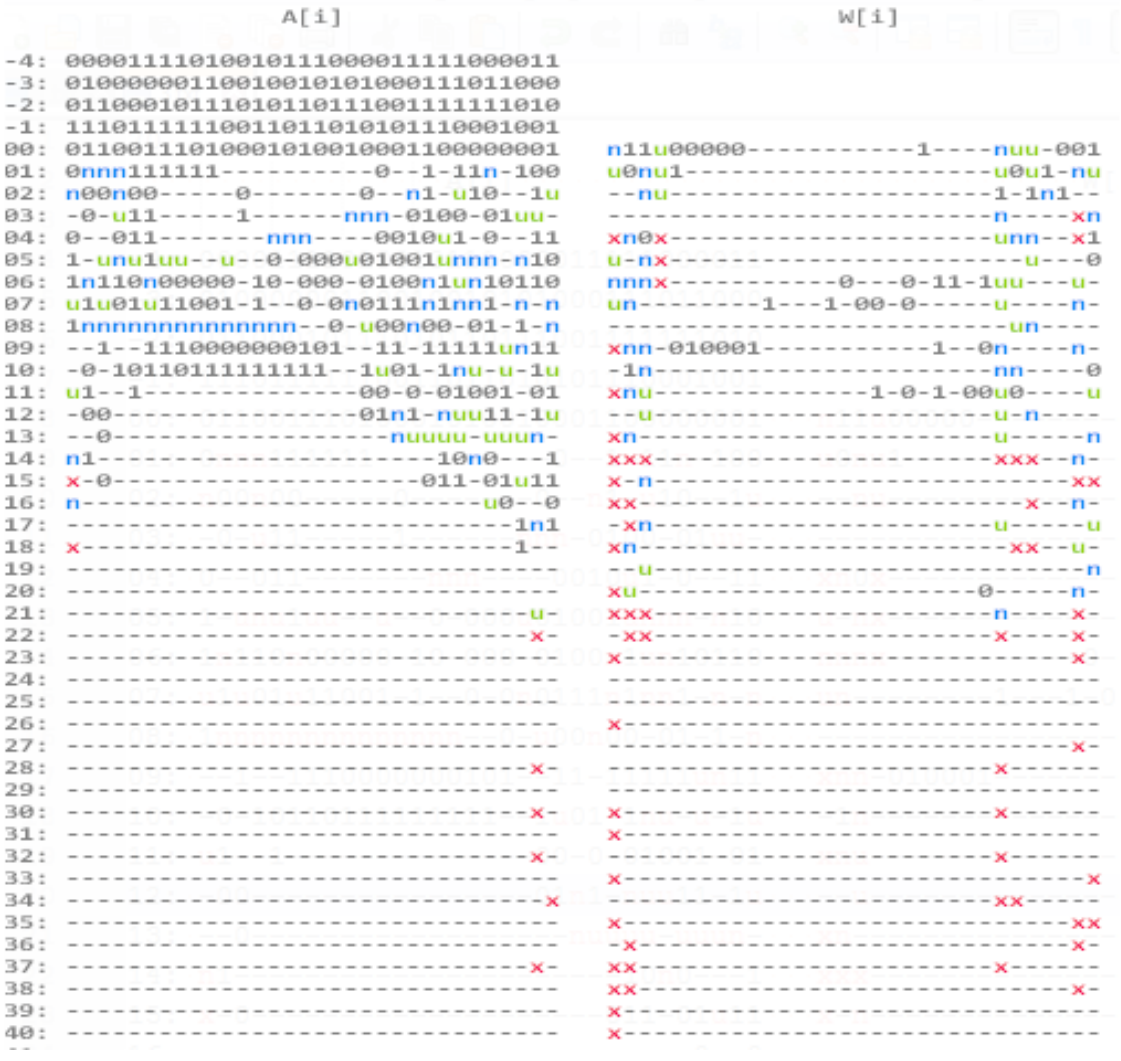

Generating a non-linear path

The non-linear path search algorithm

Input : a differential path with '?' only in the internal state in steps 1-14, with steps 14-20 being determined pseudo-linearly

Output : a differential path where no '?' nor 'x' exist anymore in the internal state : the path is fully determined and signed

- **very sensitive to many parameters**, **quite technical** and **hard to make it right**
- heuristic strategy [CR06] or using SAT solvers [SBKAM17]

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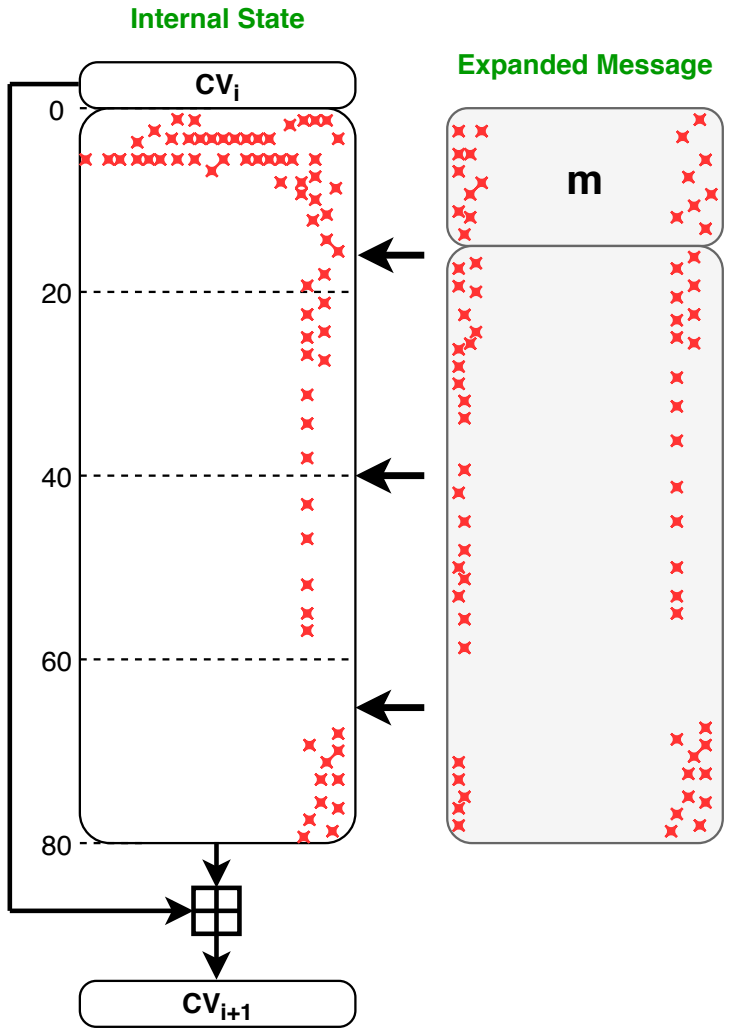


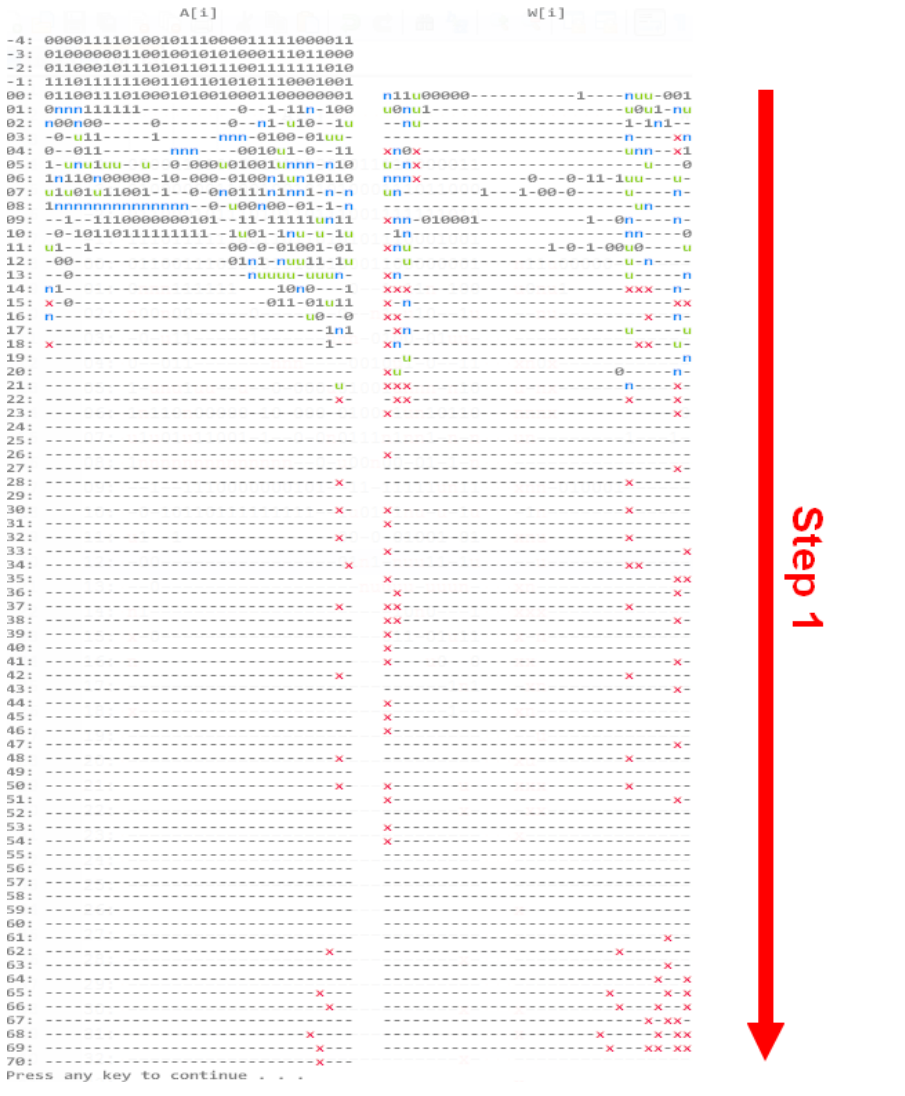
A simple collision search algorithm

A naive collision search algorithm

Repeat until a collision is found :

- pick a random message
- test if it follows the entire differential path

The **very costly** non-linear part has to be paid ☹ ☹ ☹



A better collision search algorithm

A less-naive search algorithm :

$i = 1$

repeat until a collision is found :

- pick a random message word M_i (backtrack sometimes)
- test if it follows the differential path for step i
  - if it does, then $i = i + 1$
- when $i = 16$, test if it follows the entire differential path

The very costly non-linear part is avoided ☺ ☺ ☺, **only the linear probabilistic part remains to be paid**

Final collision search algorithm

Final collision search in 3 phases :

- **Step 1 :** handling the low-probability non-linear parts using the message block freedom
- **Step 2 :** apply the collision search speed-up techniques
- **Step 3 :** the remaining steps are verified probabilistically

Computation cost further reduced 😄 😄 😄 only steps ˜22-80 have to be considered

Collision search speed-up techniques

Several techniques to speed-up the collision search :

- message modifications [WYY05]
- neutral bits [BC04]
- boomerangs (or tunnels) [K06,JP07]

All these techniques trade message freedom degrees for a speed-up factor

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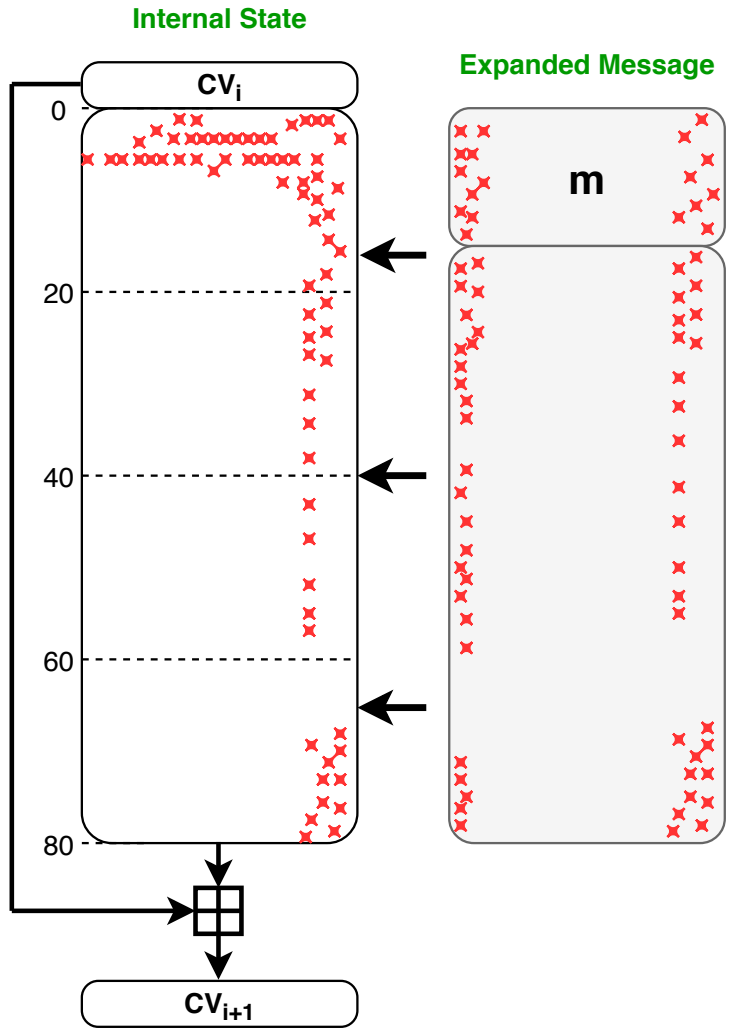
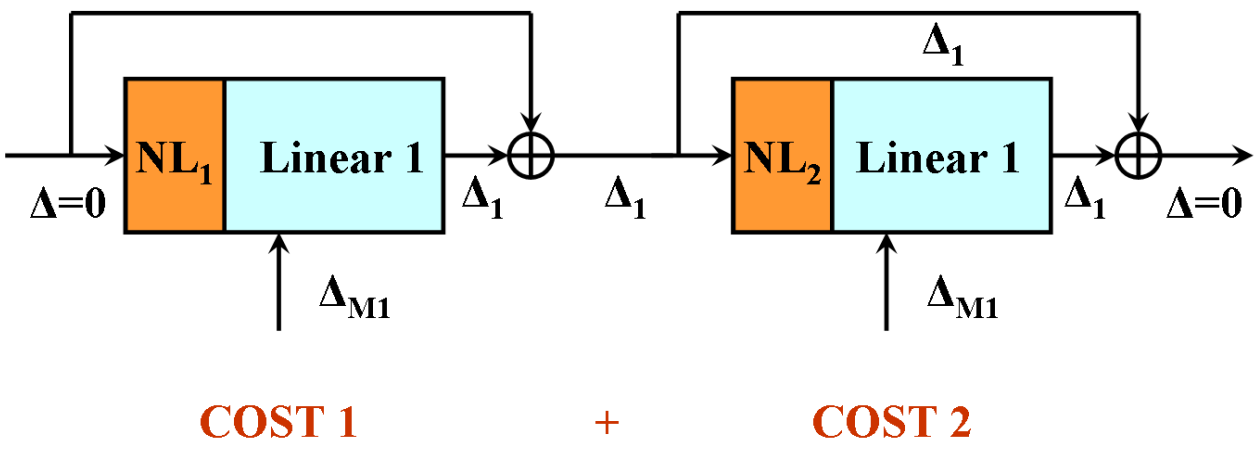


Outline

Why chosen-prefix collision is hard for SHA-1

Can the SHA-1 collision attack be directly adapted for chosen-prefix scenario ?

No : we can't remove the random difference on the chaining variable with the very small number of possible output differences of the linear path



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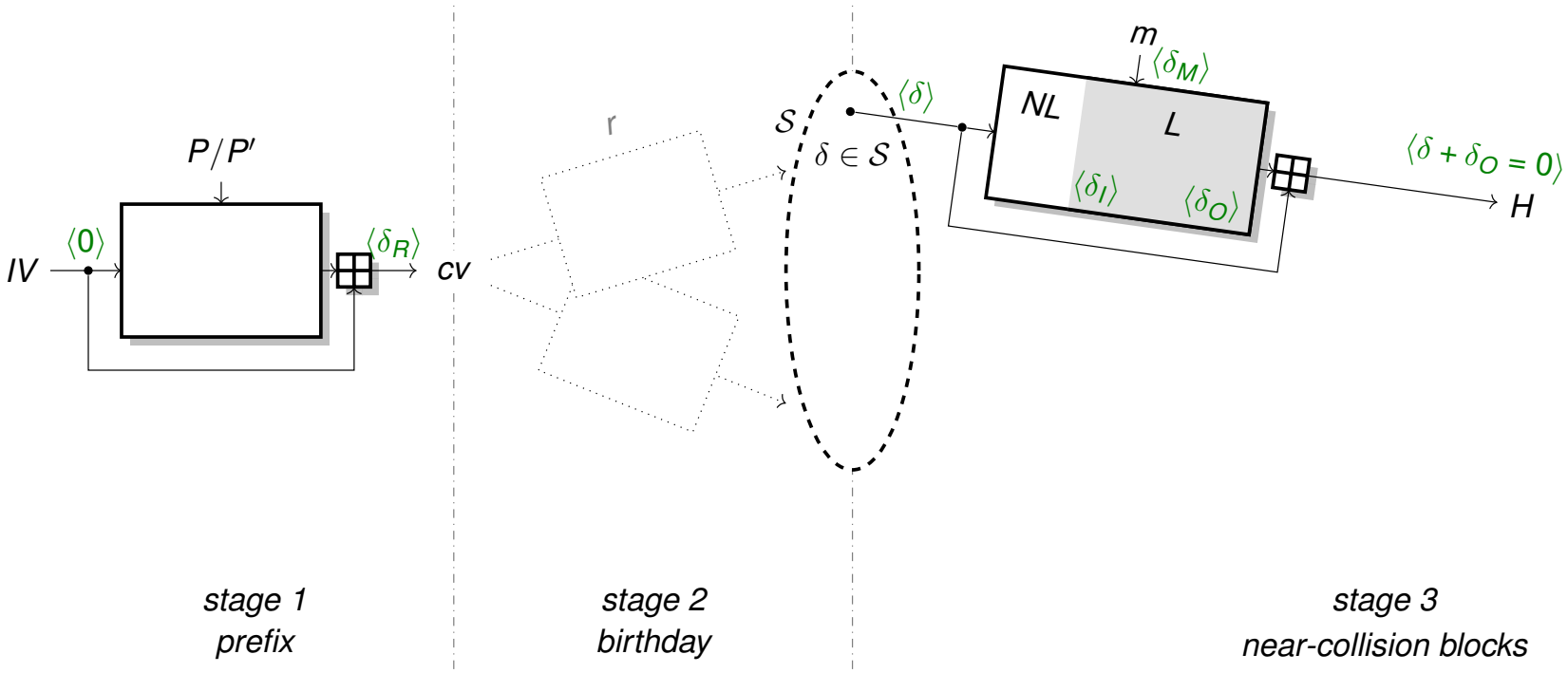
We will use the recent SHA-1 collision attack [SBKAM17] as a black box. Denote C ($\simeq 2^{64.7}$) the computational cost for the last block. (**New** : we recently reduced this cost to $2^{61.4}$)

Assume that we can use the same attack, for the same cost C, whatever is the input difference (this is possible thanks to the non-linear search algorithm). We validated this assumption in practice for several randomly chosen input differences.

Birthday to the rescue !

Trick 1 : birthday search

Use **birthday search** to reduce the entropy of possible chaining variable differences



Birthday to the rescue !

Trick 1 : birthday search

Use **birthday search** to reduce the entropy of possible chaining variable differences

Assume that you have a set $\mathcal{S}$ of differences that you can reach on the output of the internal cipher, for a computation cost C.

phase 1 : compute the (random) state difference δ_R induced by the challenge prefix

phase 2 : apply birthday strategy to map difference δ_R to a difference δ that belongs to a certain set $\mathcal{S}$ (requires $\sqrt{\pi \cdot 2^n/|\mathcal{S}|}$ computations)

phase 3 : apply the collision attack as explained previously (with a cost C) to map difference δ to a pair of colliding states

Birthday to the rescue !

Trick 1 : birthday search

Use **birthday search** to reduce the entropy of possible chaining variable differences

Stevens [S13] identified 192 possible output differences that can be reached for the same minimal cost C

phase 1 : $O(1)$

phase 2 : a birthday phase of $2^{77.1}$ computations

phase 3 : a collision phase of $C = 2^{64.7}$ computations

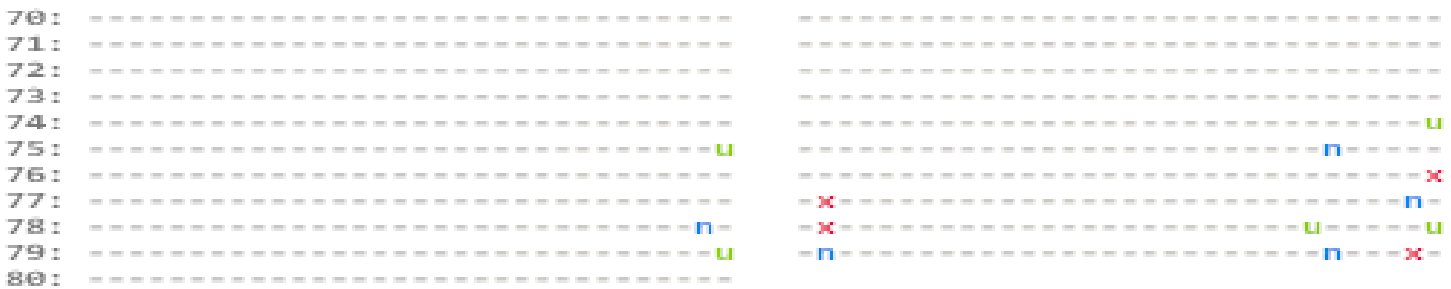
Total is $2^{77.1}$ computations (birthday phase is dominating)

Relaxing the output differences constrains

Trick 2a : generalized output differences

Using heuristics, we found **more allowable output differences** than previously known (576 instead of 192) for a cost C. This will increase $\mathcal{S}$.

For a maximal computational cost of C per block, we found a set $\mathcal{S}$ of 576 elements (instead of 192)

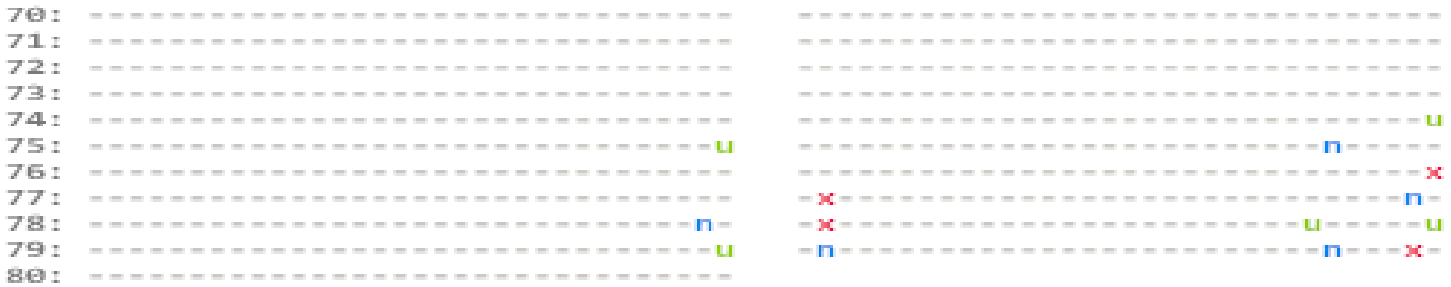


Relaxing the output differences constrains

Trick 2b : generalized output differences

Accept **more costly differential paths** to further increase $\mathcal{S}$ (so that the birthday phase and collision phase have about the same computational cost).

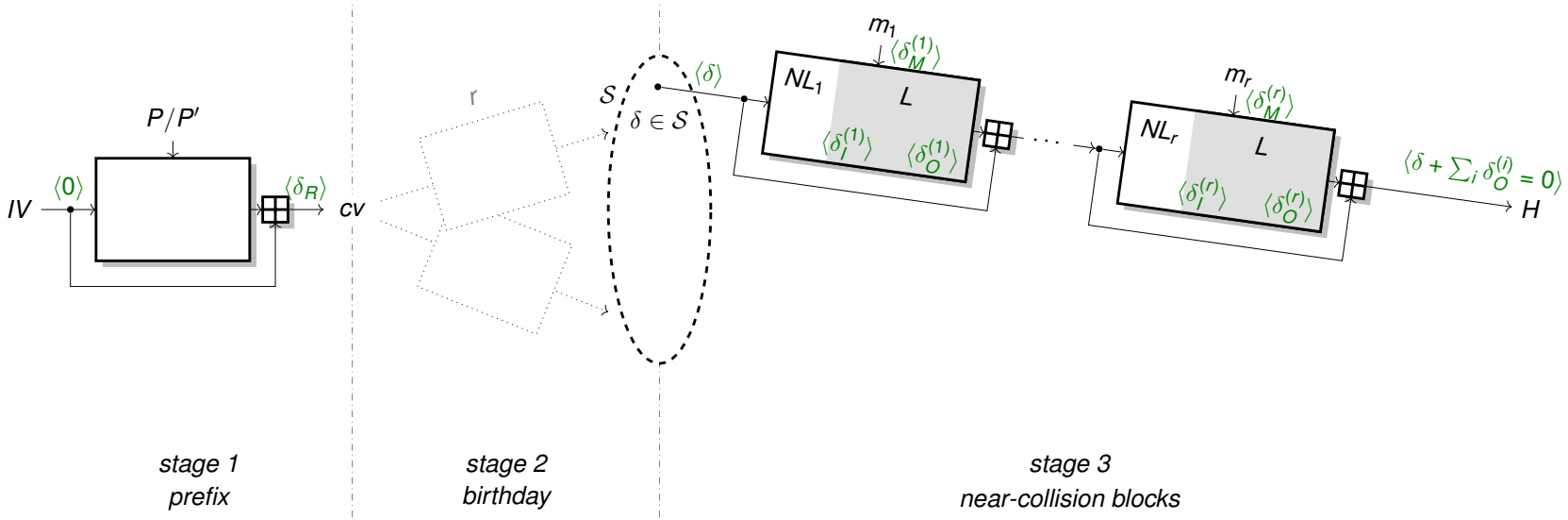
For a maximal computational cost of $8 \cdot C$ per block, we found a set $\mathcal{S}$ of 8768 elements.



Multiblocks : the more the merrier !

Trick 3 : multiblocks

Use **several blocks** to reach the collision after the birthday phase. It will increase the size of $\mathcal{S}$ and thus reduce the birthday phase cost.



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Trick 3 : multiblocks

Use **several blocks** to reach the collision after the birthday phase. It will increase the size of $\mathcal{S}$ and thus reduce the birthday phase cost.

With a maximum cost of $3 \cdot C$, one obtains a set $\mathcal{S}$ containing about 2^{30} elements !

The maximum length of the chain of blocks is 54 and the average is 17.

Multiblocks : the more the merrier !

Trick 3 : multiblocks

Use **several blocks** to reach the collision after the birthday phase. It will increase the size of $\mathcal{S}$ and thus reduce the birthday phase cost.

A three-phase attack :

phase 1 : compute the (random) state difference δ_R induced by the challenge prefix

phase 2 : apply birthday strategy to map difference δ_R to a difference δ that belongs to a certain set $\mathcal{S}$

phase 3 : apply a few consecutive SHA-1 block attacks to slowly map difference δ to a pair of colliding states

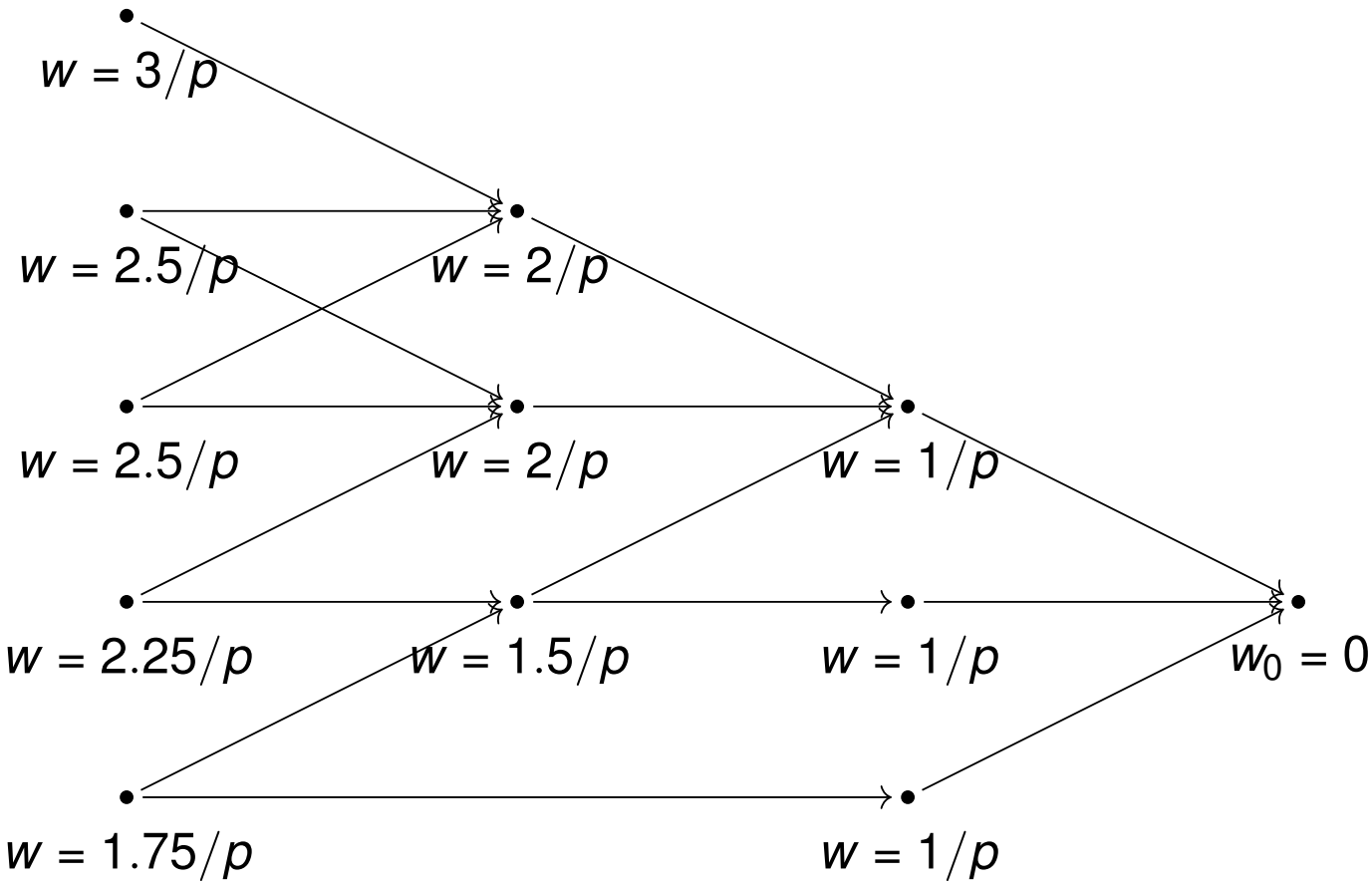
The clustering effect

Trick 4 : use the **clustering effect**

The attacker can sometimes **target several nodes simultaneously** to reduce the cost (because it is easier to hit one node out of many than a fixed one).

He will select **dynamically** the allowable differences at the output of each successive blocks. For that, we need a “map” of the whole situation, so we can decide which output difference I should be targeting at each new block. We will build a graph $\mathcal{G}$ for that.

Example of a graph $\mathcal{G}$



(we assume that all the edges have probability p)

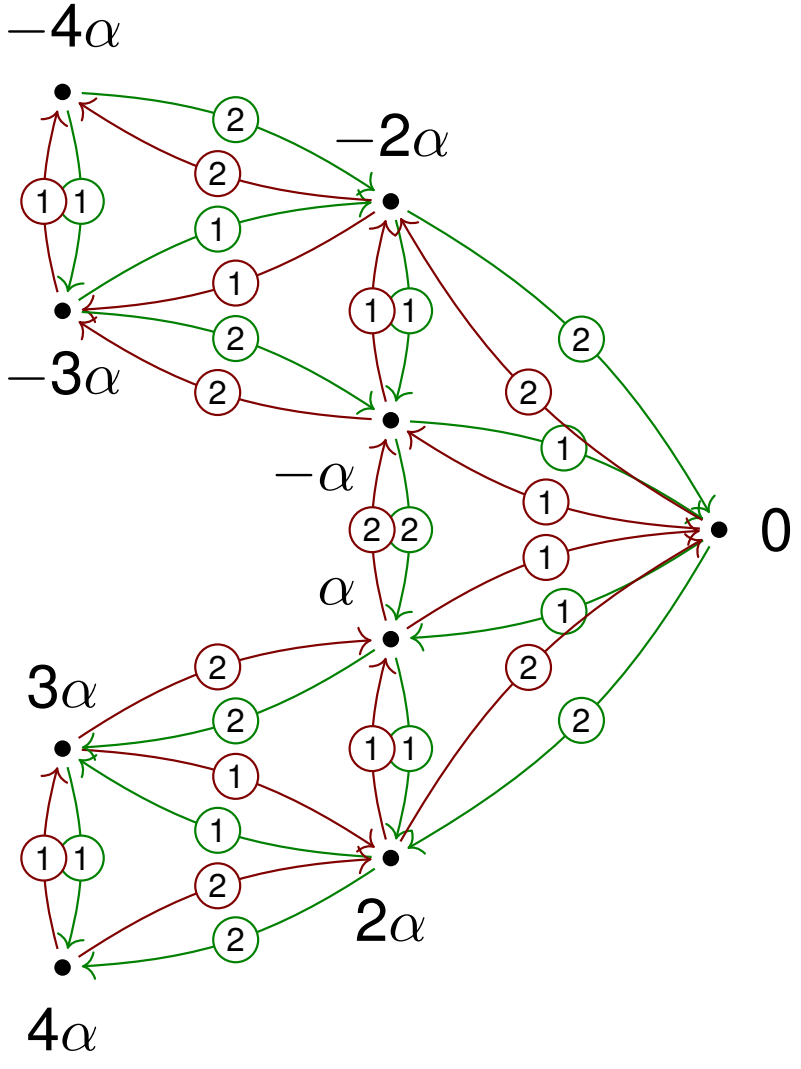
Building the graph $\mathcal{G}$ and the set $\mathcal{S}$

We call **bundle** a set of output differences that can be tried at the same time. We can build a graph $\mathcal{G}'$ with the bundles.

Consider that you have :

- a bundle $\{\alpha, 2\alpha\}$ with costs 1 and 2 (green lines)
- a bundle $\{-\alpha, -2\alpha\}$ with costs 1 and 2 (red lines)

The corresponding set $\mathcal{S}$ is $\{-4\alpha, -3\alpha, -2\alpha, -\alpha, 0, \alpha, 2\alpha, 3\alpha, 4\alpha\}$



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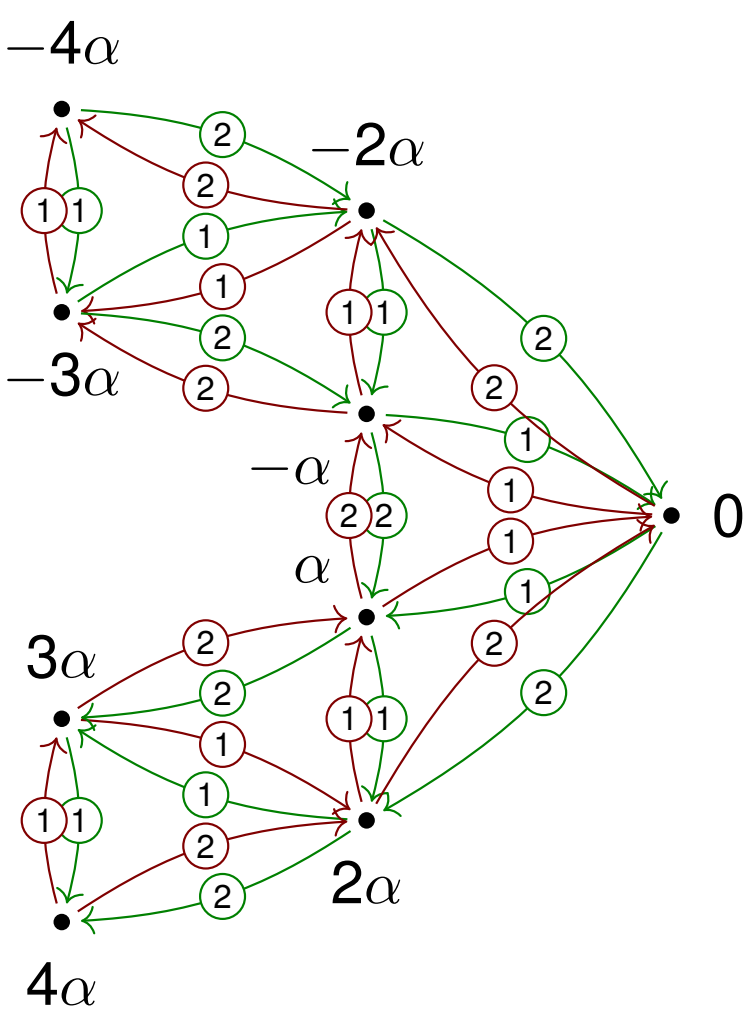

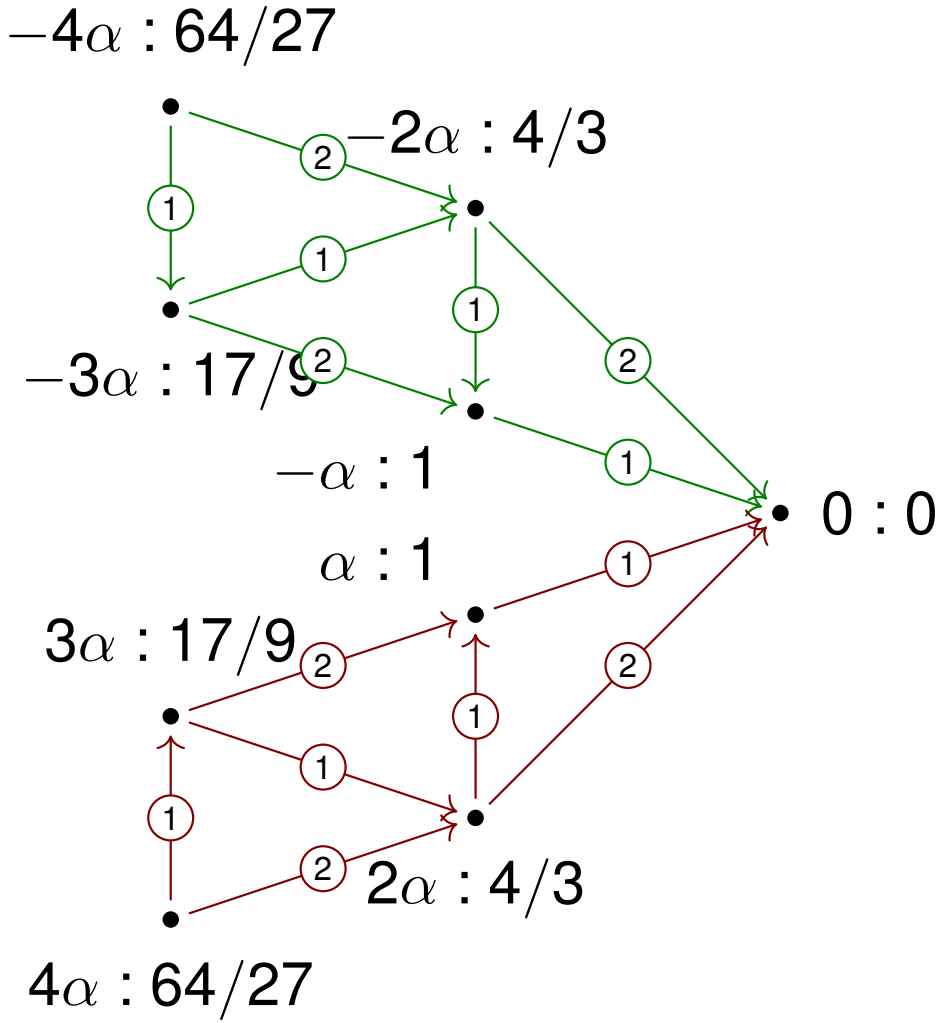

Outline

Our results : SHA-1

We obtain a **practical chosen-prefix coll. attack on** SHA-1 :

- with only $2^{63.7}$ computations
- **only a small factor more costly than a classical collision attack** (that we improved from $2^{64.7}$ to $2^{61.4}$)
- We computed an actual chosen-prefix collision

Function	Collision type	Complexity	Reference
SHA-1	free-start collision	$2^{57.5}$	[SKP16]
	collision	2^{69}	[WYY05]
		$2^{64.7}$	[SBKAM17]
		$2^{61.4}$	our result NEW
	chosen-prefix collision	$2^{77.1}$	[S13]
		$2^{67.2}$	our result EC19
		$2^{63.7}$	our result NEW

GPU renting service

Bitcoin rise/fall created cheap GPU renting services :-)

We found very cheap GPU renting service, much cheaper than Amazon services : https ://www.gpuserversrental.com/
(we rented about 900 GTX-1060 for 2 months)

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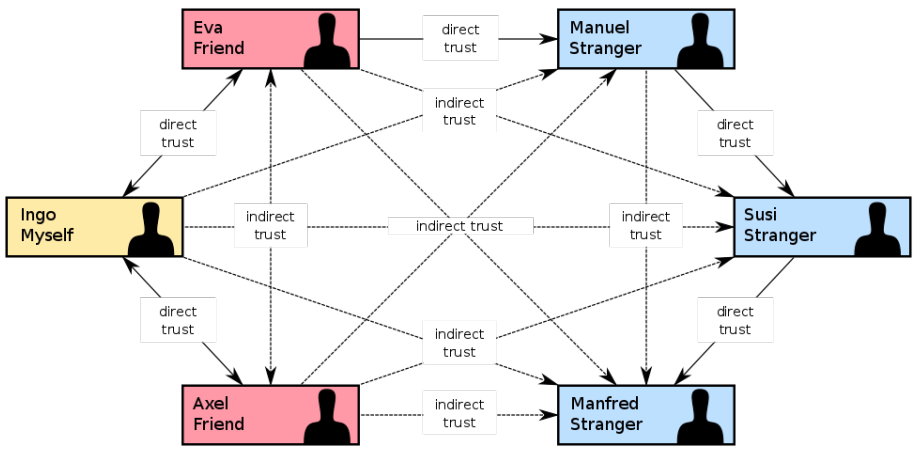
Finding the chosen-prefix collision costed us ”only” 75k USD (should be now 50k USD), classical collision costs now 10k.
In a few years, this will cost only a few thousands of USD.
Note : it was already practical since many years ago.

Breaking PGP Web-of-Trust (1)

Are we creating a simple chosen-prefix collision for SHA-1 ? No ! We have several possible proof-of-concept we could try (obvious one : creation of a rogue X.509 certificate).

CONFIDENTIAL :

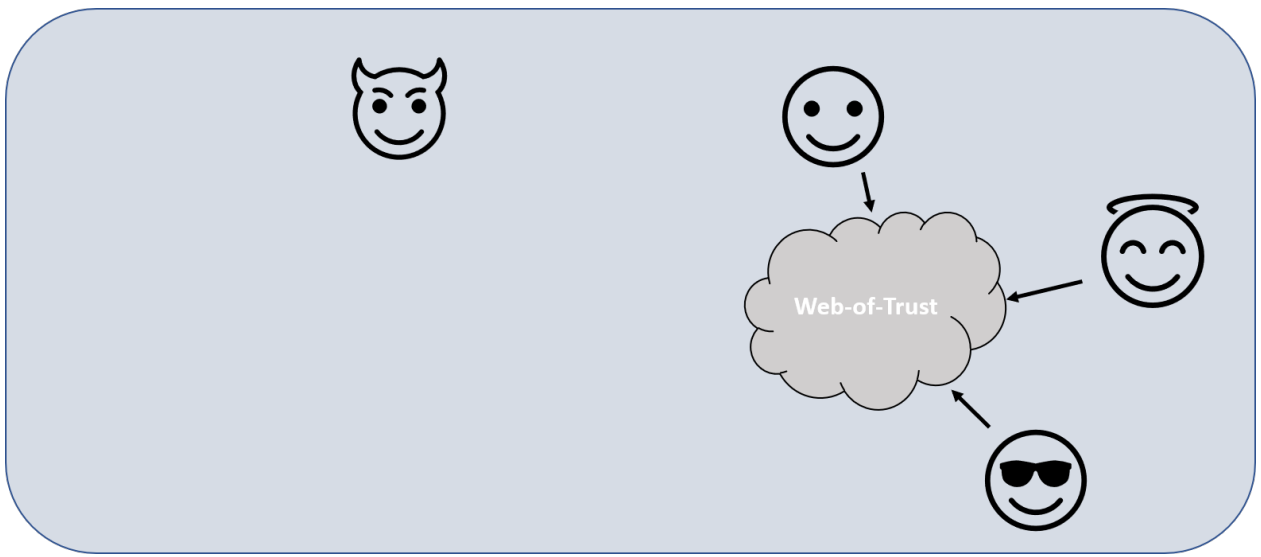
We chose to use the chosen-prefix collision to break OpenPGP key certification signatures, undermining the PGP Web-of-Trust.



Breaking PGP Web-of-Trust (2)

Idea :

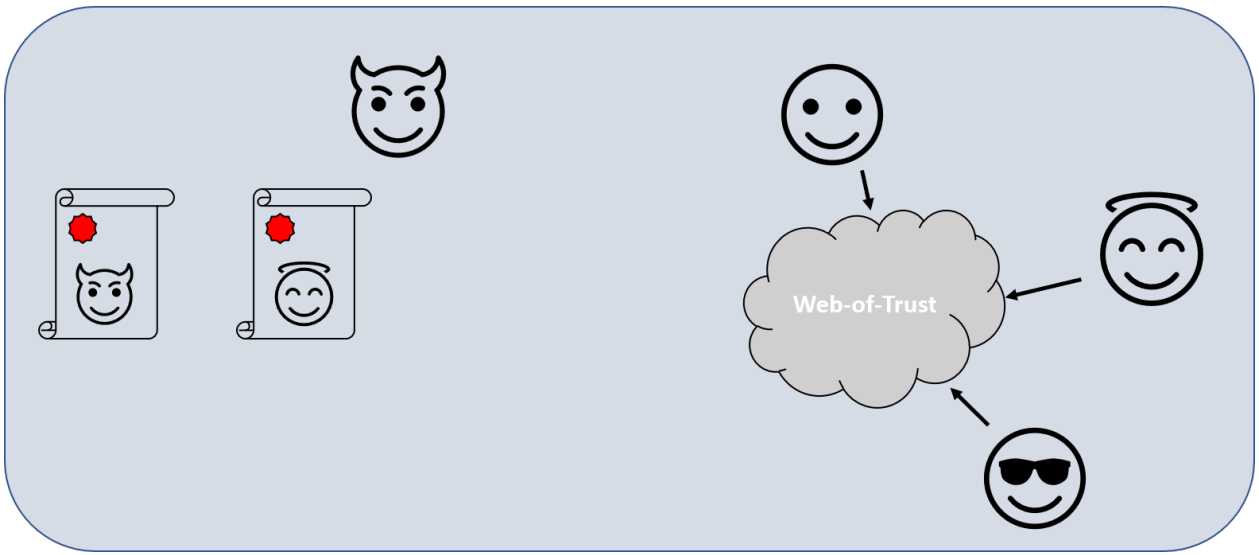
-
-
-
-



Breaking PGP Web-of-Trust (2)

Idea :

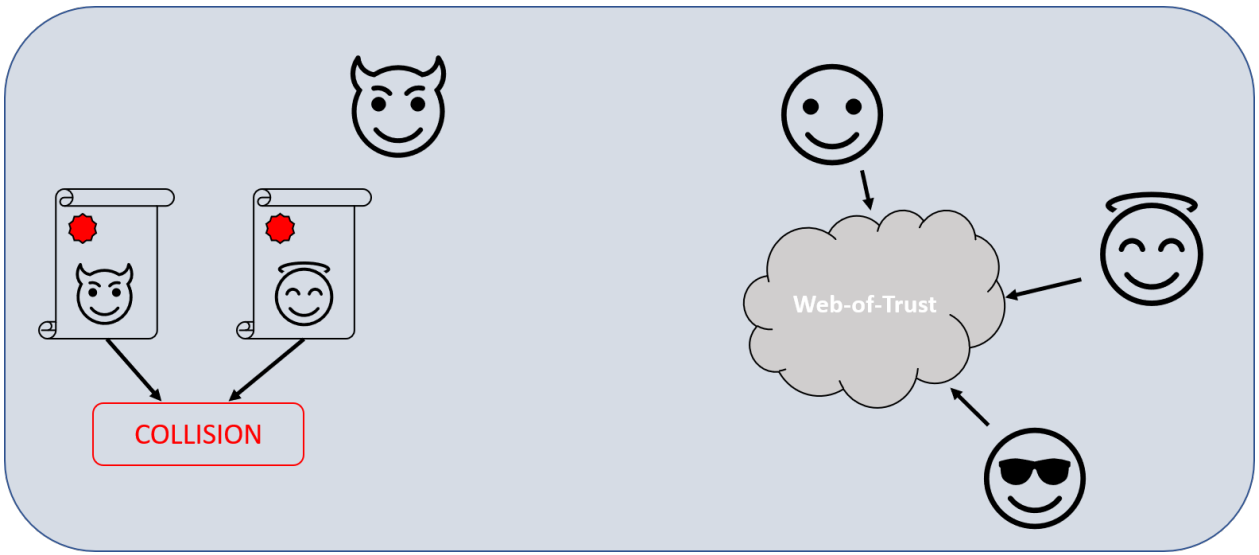
- create a pair of keys with two different UserIDs :
  one with victim name (A), and with real attacker's name (B)
-
-
-



Breaking PGP Web-of-Trust (2)

Idea :

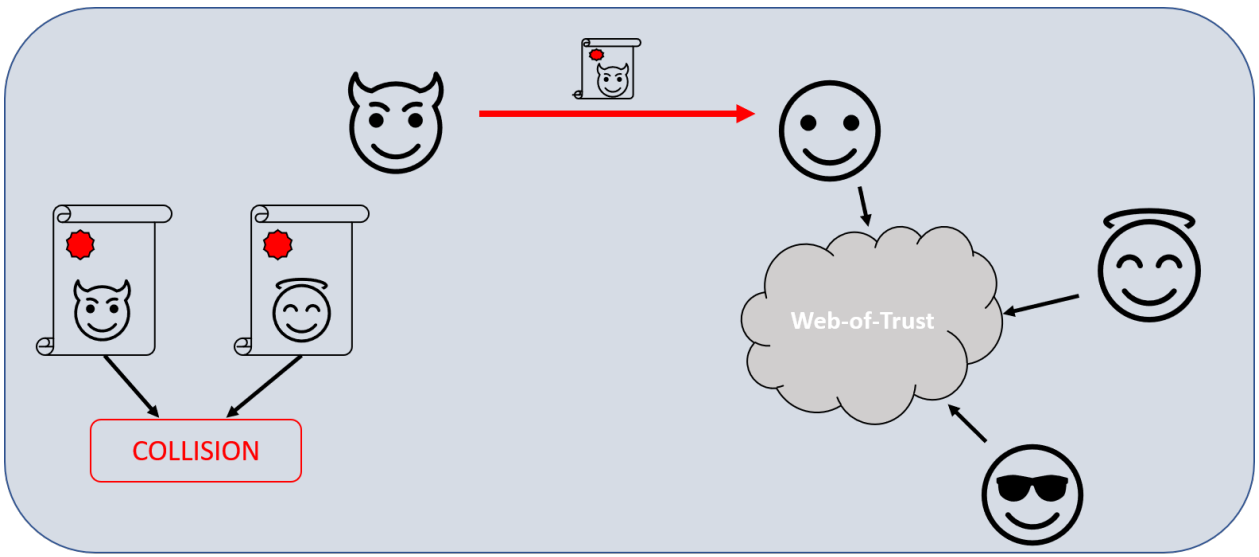
- create a pair of keys with two different UserIDs
- using a chosen-prefix collision, we craft the keys such that the SHA-1 hash that is signed for the key certification is the same for both keys.
-
-



Breaking PGP Web-of-Trust (2)

Idea :

- create a pair of keys with two different UserIDs
- collide key certifications
- the attacker asks for key certifications of key B : since he knows the corresponding secret key, and the UserID matches his official ID, he will collect trust-worthy signatures and integrate the web-of-trust.
-



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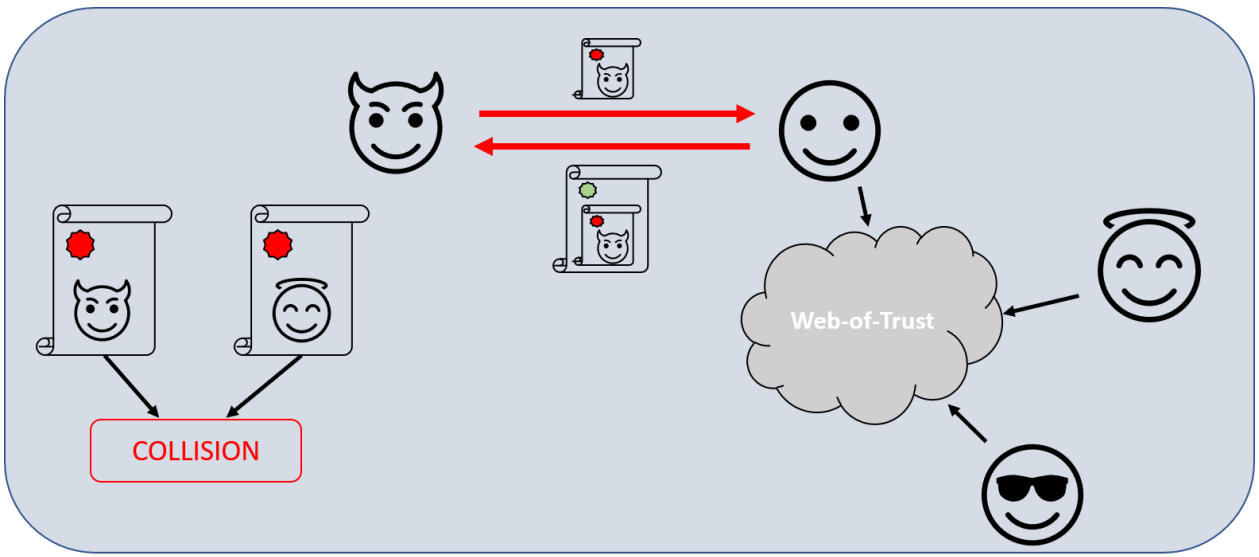
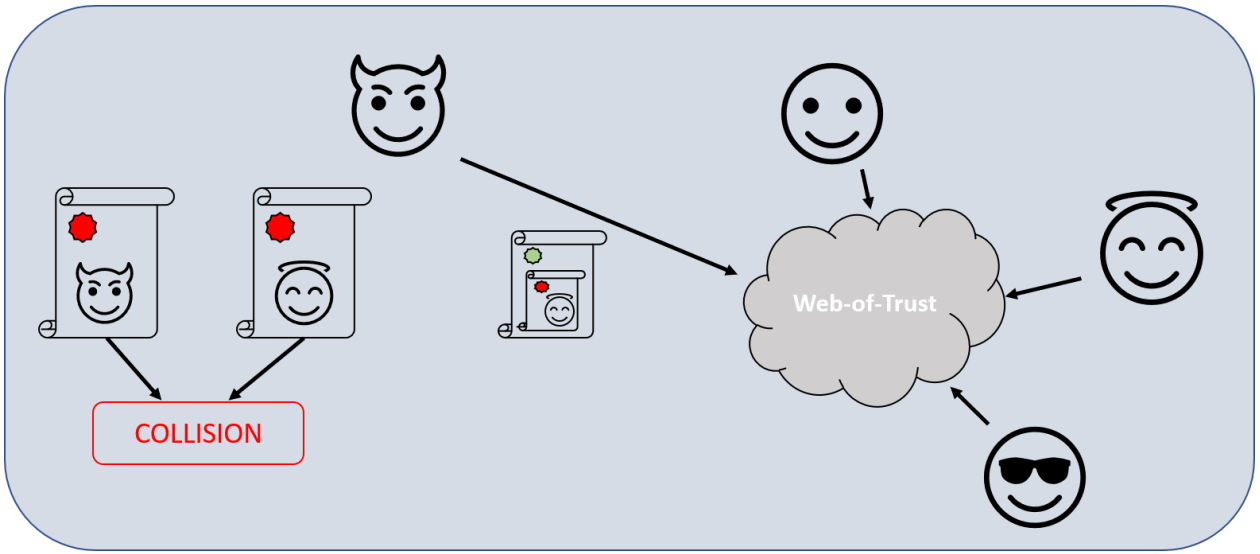


Breaking PGP Web-of-Trust (2)

Idea :

- create a pair of keys with two different UserIDs
- collide key certifications
- integrate web of trust with UserID B
- since the hash of both keys collide, he can transplant the signatures to key A, creating a key with the UserID of the victim, trusted by the web-of-trust, and for which he controls the secret key. He can then sign messages pretending to be the victim.



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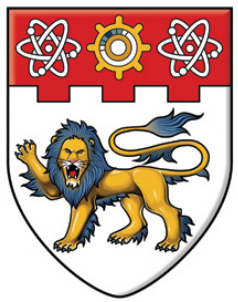
This result will be announced at RWC 2020

Thank you for your attention !



NANYANG TECHNOLOGICAL UNIVERSITY