Chosen-Prefix Collision Attacks on SHA-1

Thomas Peyrin

(joint work with Gaëtan Leurent)

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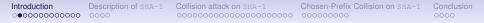
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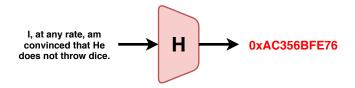
Outline

Introduction

Finding a non-linear differential part



What is a Hash Function?



• *H* maps an **arbitrary length input** (the message *M*) to a **fixed length output**. Typically *n* = 128 (MD5), *n* = 160 (SHA-1) or *n* = 256 bits (SHA-256).

- no secret parameter.
- *H* must be easy to compute.

The security goals

pre-image resistance :

given an output challenge *y*, the attacker can not find a message *x* such that H(x) = y, in less than $\theta(2^n)$ operations.

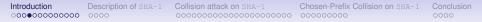
2nd pre-image resistance :

given a challenge (x, y) so that H(x) = y, the attacker can not find a message $x' \neq x$ such that H(x') = y, in less than $\theta(2^n)$ operations.

collision resistance :

the attacker can not find two messages (x, x') such that H(x) = H(x'), in less than $\theta(2^{n/2})$ operations (a generic attack with the birthday paradox exists [Yuval-79]).

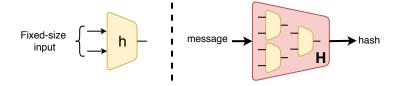
$$x \rightarrow H \rightarrow ? \leftarrow H \leftarrow x'$$



General hash construction

For historical reasons, most hash functions are composed of two elements :

- a compression function *h* : a function for which the input and output size is fixed.
- a domain extension algorithm : an iterative process that uses the compression function *h* so that the hash function *H* can handle inputs of arbitrary length.



The Merkle-Damgård domain extension algorithm

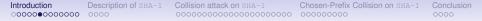
The most famous domain extension algorithm used is called the Merkle-Damgård [Merkle Damgård-89] iterative algorithm.

M₁ Ma M₃ Mn h h h h

The compression function h now takes two fixed-size inputs, the incoming chaining variable cv and the message block m, and outputs a new chaining variable.

 $pad(M) = M_1 \parallel M_2 \parallel M_3 \parallel ... \parallel M_n$

+ hash



Compression function security notions

Idea : a collision on an iterated hash function *H* always comes from a collision on the compression function *h* :

$$H(M) = H(M^*) \Longrightarrow h(cv, m) = h(cv^*, m^*)$$

The conditions on (cv, m) give different kind of attacks

Collision $cv = cv^*$ fixed and $m \neq m^*$ free.

Semi-free-start Collision $cv = cv^*$ and $m \neq m^*$ are free.

Free-start Collision $(cv, m) \neq (cv^*, m^*)$ are free.

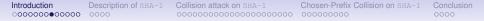
If the compression function is broken, go away from this hash function as soon as possible !

The sad story of MD5

The cryptanalysis history of MD5 is a good example of why (semi)-free-start collisions are a serious warning.

1992 MD5 RFC published

- 1993 pseudo-collision on the compression function [BB93]
- 1994 semi-free-start collision on the compression function [Dob96]
- 2004 practical collisions on the hash function [WFL04]
- 2007 chosen-prefix collisions and colliding X.509 certificates [SLW07]
- 2009 rogue CA certificates for RapidSSL [S+09] (used chosen-prefix collision)
- 2010-2012 Flame malware (used chosen-prefix collision)



Current state of MD-SHA-family

MD-SHA-family encompasses many hash functions :

1990' MDx (MD4, MD5, SHA-0, SHA-1, HAVAL, RIPEMD) **2002** SHA-2 (SHA-224, ..., SHA-512)

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Some old hash functions are still unbroken :

Broken MD4, MD5 Broken SHA-0, SHA-1 Broken HAVAL Broken RIPEMD-0 Unbroken RIPEMD-128, RIPEMD-160 Unbroken SHA-2

What about SHA-1?

The current situation of SHA-1 :

1995 SHA-1 NIST FIPS 180-1 published : basically SHA-0 (1993) with a very small twist

2005 theoretical collision attack on the full hash function [WYY05] - 2^{69}

- 2006-2011 lots of works computing actual collisions for reduced-round versions of SHA-1
 - 2015 free-start collision on the full compression computed [SKP15] - 2⁵⁷
 - 2017 full collision on the full hash function computed [SBKAM17] 2^{64.7}

- ??? chosen-prefix collision attack?
- ??? SHA-1 collisions in the wild?



Motivations to study SHA-1

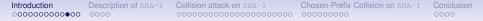
Why still studying SHA-1?

Design from NSA, **Standard** from NIST, **Still used worldwide** despite deprecation efforts (major browsers now refusing to connect to servers still using SHA-1- based certificates) :

- more than 5% of Alexa's top 1 million prefer to use SHA-1 to authenticate TLS handshake messages (including www.skype.com)
- about 30000 servers with SHA-1 certificates (out of 720000 servers with HTTPS support)
- other protocols : about 1 million out of 4.55 millions mail servers (with IMAPS) use a <code>SHA-1</code> certificate
- it is still possible to buy a SHA-1 certificate from a trusted root, and some can be found in the wild
- the "Mail" application included in Windows 10 still accepts SHA-1 certificates without warnings when opening an IMAPS connection

Yet another push is perhaps needed to accelerate the retirement of SHA-1

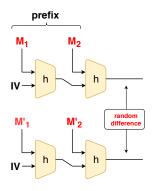
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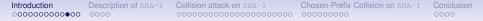


What are chosen-prefix collisions?

Chosen-prefix collision attack

The attacker is first challenged with **two message prefixes** *P* and *P'*, and its goal is to compute two messages *M* and *M'* to create the **collision** H(P||M) = H(P'||M'), where || denotes concatenation

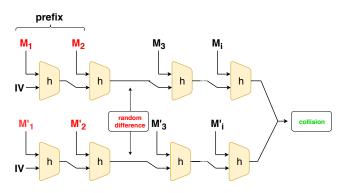




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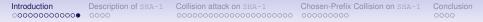
- **much more powerful** than a simple collision attack (i.e. rogue CA certificate)
- supposedly much harder than a simple collision attack (currently for MD5, from 2¹⁶ to 2³⁹)
- birthday attack can apply, thus generic cost remains 2^{n/2} (i.e. 2⁸⁰ in the case of SHA-1)

Why chosen-prefix collisions?

Colliding SSL certificates [S+09] :

serial number	chosen prefix (difference)	serial number
validity period		validity period
real cert domain name		rogue cert domain name
real cert RSA key	collision bits (computed)	real cert RSA key
X.509 extensions	identical bytes (copied from real cert)	X.509 extensions
signature		signature

Alex Sotirov https://trailofbits.files.wordpress.com/2012/06/flame-md5.pdf



Chosen-prefix collisions for SHA-1

Current status of chosen-prefix collisions on SHA-1 :

- current best known chosen-prefix attack against SHA-1 requires 2^{77.1} computations [S13] (a factor 8 better than generic attack).
- ... while classical collision could be found with 2^{64.7} computations.
- one can't apply directly the SHA-1 collision attack, because of the random state difference due to the challenge prefix.

Can we reduce the gap and make chosen-prefix collisions practical for SHA-1?

Description of SHA-1

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Outline

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Description of SHA-1

Collision attack on SHA-1

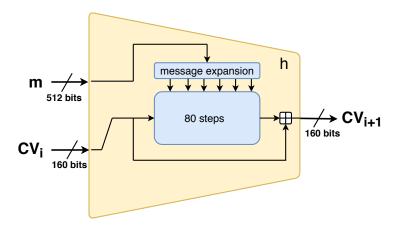
Finding a linear differential path Finding a non-linear differential part Collision search speed-up techniques Launching the collision search

Chosen-Prefix Collision on SHA-1

Conclusion

The SHA-1 compression function

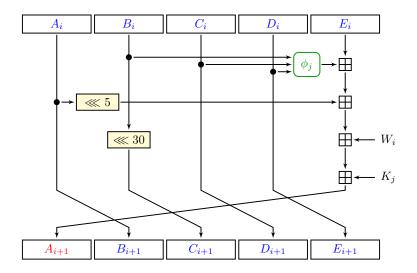
 $m = M_0 ||M_1|| \cdots ||M_{15}|$



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The SHA-1 step function



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The SHA-1 step function : alternative representation

Message expansion :

 $W_i = M_i$ for i = [1, ..., 15], then

$$W_i = (W_{i-16} \oplus W_{i-14} \oplus W_{i-8} \oplus W_{i-3})^{\ll 1}$$

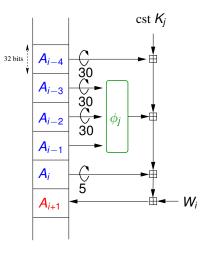
Boolean functions ϕ_j :

 $\mathsf{IF}(x, y, z) \coloneqq (x \land y) \oplus (\bar{x} \land z)$ for steps 0 to 19

 $XOR(x, y, z) := x \oplus y \oplus z$ for steps 20 to 39

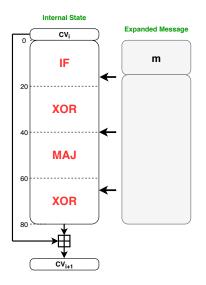
 $\mathsf{MAJ}(x, y, z) := (x \land y) \oplus (x \land z) \oplus (y \land z)$ for steps 40 to 59

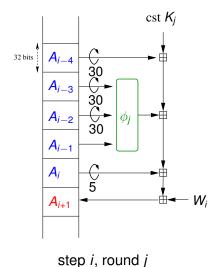
 $XOR(x, y, z) := x \oplus y \oplus z$ for steps 60 to 79



step i, round j

The SHA-1 step function : alternative representation





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Description of SHA-1

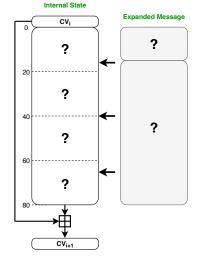
Collision attack on SHA-1 Chosen-Prefix Co

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Conclusion

Goal : Find a collision for SHA-1

- Find a linear path for rounds 16-80, using local collisions this will fix the entire message difference, and the internal state difference for steps 16-80
- Find a non-linear path for steps 1-15, using heuristic algorithm this will fix the internal state difference for rounds 1-15
- Prepare the collision search speed-up techniques by using the freedom degrees available
- Launch the collision search!



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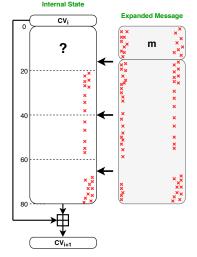
Description of SHA-1

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Chosen-Prefix Collision on SHA-1 000000000 Conclusion

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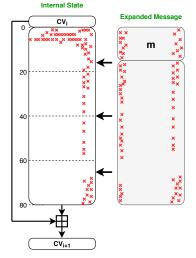
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Description of SHA-1

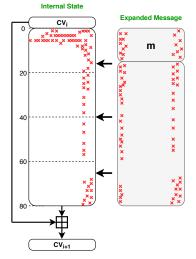
Collision attack on SHA-1 Chosen-F

Chosen-Prefix Collision on SHA-1

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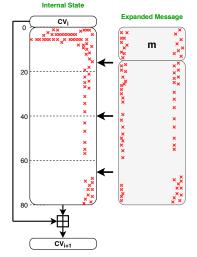
Description of SHA-1

Collision attack on SHA-1 Chosen-I

Chosen-Prefix Collision on SHA-1 000000000 Conclusion

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Collision attack on SHA-1 Finding a linear differential path

Finding a non-linear differential part Collision search speed-up techniques Launching the collision search

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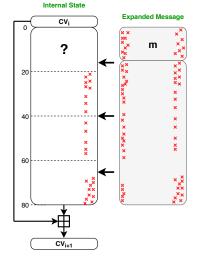
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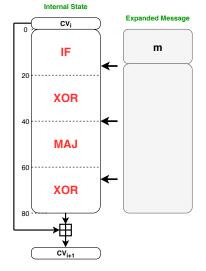


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What shape should have the differential path?

Properties of the boolean functions :

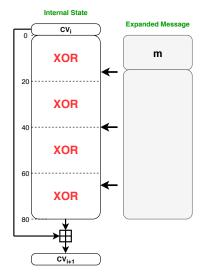
- XOR : no possible absorption of a difference
- MAJ : possible absorption of a difference (can always behave as a XOR)
- IF : possible absorption of a difference (can always behave as a XOR, except if two differences in the inputs)



What shape should have the differential path?

Idea : linearise SHA-1

- modular addition is replaced by XOR (probability 1/2 per difference bit)
- boolean functions IF and MAJ are replaced by XOR (probability depends on situation)
- all the rest is already linear !



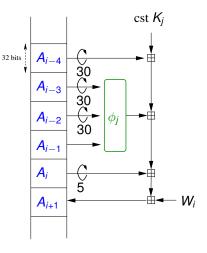
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Local collisions [CJ98]

Local collision [CJ98]

Insert a perturbation difference and **correct it** so that it doesn't spread out!



step *i*, round *j*

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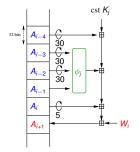
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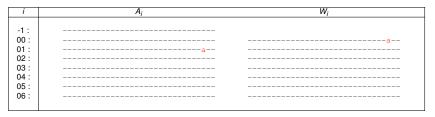
Chosen-Prefix Collision on SHA-

Conclusion 0000

Local collisions



step	type	constraints
<i>i</i> + 1	no carry	$W_j^j = a, A_{j+1}^j = a$



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 Description of SHA-1

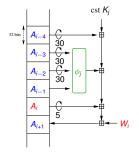
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Collision attack on SHA-1 Chosen-Prefix

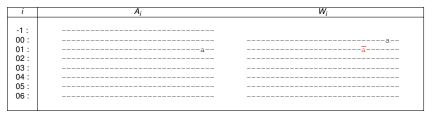
Chosen-Prefix Collision on SHA-

Conclusion 0000

Local collisions



step	type	constraints
<i>i</i> + 1	no carry	$W_j^j = a, A_{j+1}^j = a$
i + 2	correction	$W_{i+1}^{j+5} = \overline{a}$

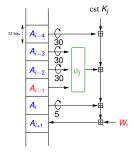


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Collision attack on SHA-1 Chosen-Prefix Collision of October Collision o

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Local collisions



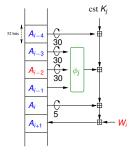
step	type	constraints
<i>i</i> + 1	no carry	$W_{j}^{j} = a, A_{j+1}^{j} = a$
i + 2	correction	$W_{i+1}^{j+5} = \overline{a}$
i + 3	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
	correction	$A_{i-1}^{j+2} \neq A_i^{j+2}, W_{i+2}^j = \overline{a}$



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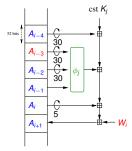
Local collisions



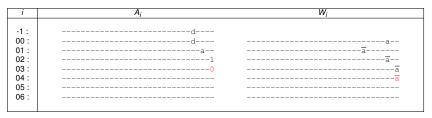
step	type	constraints
<i>i</i> + 1	no carry	$W_i^j = a, A_{i+1}^j = a$
i + 2	correction	$W_{i+1}^{j+5} = \overline{a}$
<i>i</i> + 3	correction	$A_{i-1}^{j+2} \neq A_i^{j+2}, W_{i+2}^j = \overline{a}$
i + 4	no correction	$A_{i+2}^{j-2} = 0$
	correction	$A_{i+2}^{j-2} = 1, W_{i+3}^{j-2} = \overline{a}$



Local collisions

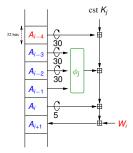


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i + 3	correction	$A_{i-1}^{j+2} \neq A_i^{j+2}, W_{i+2}^j = \overline{a}$
<i>i</i> + 4	correction	$A_{i+2}^{j-2} = 1, W_{i+3}^{j-2} = \overline{a}$
i + 5	no correction	$A_{i+3}^{j-2} = 1$
	correction	$A_{i+3}^{j-2} = 0, W_{i+4}^{j-2} = \overline{a}$

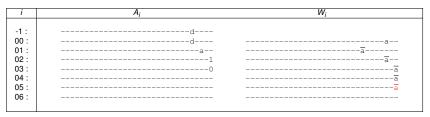


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Local collisions



step	type	constraints
<i>i</i> + 1	no carry	$W_j^j = a, A_{j+1}^j = a$
i + 2	correction	$W_{i+1}^{j+5} = \overline{a}$
i + 3	correction	$A_{i-1}^{j+2} \neq A_i^{j+2}, W_{i+2}^j = \overline{a}$
<i>i</i> + 4	correction	$A_{i+2}^{j-2} = 1, W_{i+3}^{j-2} = \overline{a}$
i + 5	correction	$A_{i+3}^{j-2} = 0, W_{i+4}^{j-2} = \overline{a}$
<i>i</i> + 6	correction	$W_{i+5}^{j-2} = \overline{a}$



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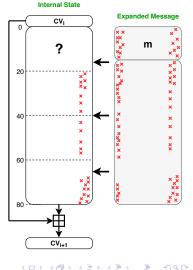
Collision attack on SHA-1 Chosen-Prefix Collision on SHA

Conclusion 0000

Linearisation of SHA-1

How to build a linear differential path :

- use only local collisions
- the mask of local collisions simply needs to be generated from the message expansion formula
- this will fix all the differences in the expanded message and in the 60 last steps of the internal state



Introduction Description of SHA

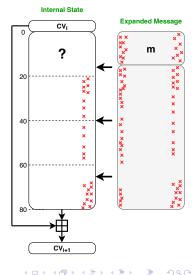
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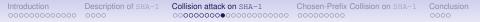
Conclusion

Linearisation of SHA-1

Finding the best linear path candidate :

- a local collision costs is 2⁻⁴ (can be improved when placed at the proper bit position to avoid carry conditions)
- go through all the possible low hamming weight 16-step windows (going through all the possible windows would be too costly)
- compute the cost induced by the local collisions for this candidate
- keep the best one (i.e. the cheapest) for steps 16-80 !



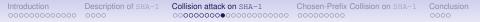


Why did we remove the 16 first steps?

it allows to avoid impossibilities due to the IF function

- impossible to get a one-block collision with good probability : necessary to use several blocks
- A non-linear part allows to start from any incoming difference in the chaining variable : 2-block collision

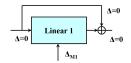
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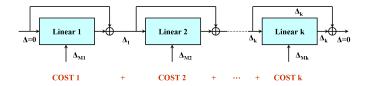
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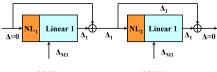


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COST 1 + COST 2

Collision attack on SHA-1

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Description of SHA-1

Collision attack on SHA-1 Finding a non-linear differential part

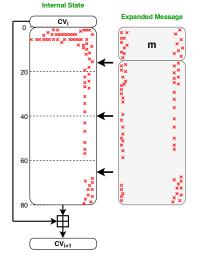
Chosen-Prefix Collision on SHA-1

Conclusion

Collision attack on SHA-1

Goal : Find a collision for SHA-1

- Find a linear path for rounds 16-80, using local collisions
- Find a non-linear path for steps 1-15, using heuristic algorithm this will fix the internal state difference for rounds 1-15
- Prepare the collision search speed-up techniques by using
- Launch the collision search !



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Collision attack on SHA-1

Example of a non-linear path

	A[1]	W[i]
-4: -3:	00001111010010111000011111000011 01000000	
-2:	01100010111010110111001111111010 1110111111	File Edit Image La
01:	???????????????????????????????????????	x-xx
03: 04:	??????????????????????????????????????	
05: 06:	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	x-xx- xxxx
07: 08: 09:	<pre>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>></pre>	xx
10:	222222222222222222222222222222222222222	××× × × × × × × × × × × × × × × × × ×
12: 13:	??????????????????????????????????????	xxxxxx
14: 15: 16:	>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>	xxx
17:	×	-xx
19: 20:		
21: 22: 23:	×	×××
24:		^^
26: 27:		xx-
28: 29: 30:	×	×x
31:		×
33: 34:	×	xx
35: 36: 37:		×
38:		××
40:		x

ntroduction Descript

Example of a non-linear path

	A[i]	W[1]
-4:	00001111010010111000011111000011	
-3:	01000000110010010101000111011000	
-2:	011000101110101101110011111111010	
-1:	11101111110011011010101110001001	
00:	01100111010001010010001100000001	n11u000001nuu-001
01:	0nnn111111101-11n-100	u0nu1u0u1-nu
02:	n00n0000n1-u101u	nu1-1n1
03:	-0-u111nnn-0100-01uu- 0011nnn0010u1-011	xn
04:		xn0xunnx1
05: 06:	1-unu1uuu0-000u01001unnn-n10	u-nx00-11-1000
07:	u1u01u11001-10-0n0111n1nn1-n-n	un11-00-0u
07:08:	1001011001-10-00011101001-0-0 100000000	unun
00:	1111000000010111-11111un11	xnn-01000110nn-
10:	-0-10110111111111-1u01-1nu-u-1u	-1n0
11:	u1100-0-01001-01	xnu1-0-1-00u0u
12:	-0001n1-nuu11-1u	xnu
13:	0nuuuu-uuun-	Xnn
14:	n110n01	xxx
15:	x-0011-01u11	x-n
16:	n	xxx
17:	1n1	-xn
18:	×1	xnxxu-
19:		n
20:		xu00
21:	u-	xxxx-
22:	×-	-xxx
23:		xx-
24:		
25:		
26:		×
27:		×-
28:	×-	×
29:		
30:	x-	xx
31:		×
32:	×-	××
33: 34:		xx
34:	×	×××
36:		×
36:		-xx-
37:	××	xxx-
39:		×**
40:		× · · · · · · · · · · · · · · · · · · ·

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Generating a non-linear path

The non-linear path search algorithm

Input : a differential path with '?' only in the internal state in steps 1-14, with steps 14-20 being determined pseudo-linearly

Output : a differential path where no '?' nor 'x' exist anymore in the internal state : the path is fully determined and signed

- very sensitive to many parameters, quite technical and hard to make it right
- heuristic strategy [CR06] or using SAT solvers [SBKAM17]

ntroduction Description of SHA-

Collision attack on SHA-1 Chosen

Chosen-Prefix Collision on SHA

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Conclusion 0000

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Chosen-Prefix Collision on SHA-1

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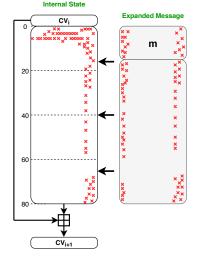
Description of SHA-1

Collision attack on SHA-1 Chosen-Pr

Chosen-Prefix Collision on SHA-1 000000000 Conclusion

Goal : Find a collision for SHA-1

- Find a linear path for rounds 16-80, using local collisions this will fix the entire message difference, and the internal state difference for steps 16-80
- Find a non-linear path for steps 1-15, using heuristic algorithm this will fix the internal state difference for rounds 1-15
- Prepare the collision search speed-up techniques by using the freedom degrees available
- Launch the collision search !



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ntroduction Description of SE

Collision attack on SHA-1 Cha

Chosen-Prefix Collision on SHA-: DOOOOOOOO Conclusion

Step

A simple collision search algorithm

A naive collision search algorithm

Repeat until a collision is found :

- pick a random message
- test if it follows the entire differential path

The **very costly** non-linear part has to be paid C C C

-4:	00001111010010111000011111000011	
- 2.1	0100000110010010101000111011000	
-21	011000101110101101110011111111010	
-11	111011111100110110101011110001001	
00:	01100111010001010010001100000001	n110000001nuu-001
811	0nnn111111101-11n-100	u0nu1
82:	n00n0000n1-u101u	
0.2 -	-0-u111nnn-0100-01uu-	
841	0-01100100-0100-0100-0100-0100-01	YNOY
84:	00110-000001-011 1-unu1uuu0-000001001unnn-n10	u-nx
051		nnnx
	1n110n00000-10-000-0100n1un10110	nnnx00-11-1uuu-
87:	u1u01u11001-10-0n0111n1nn1-n-n	un11-00-0
98:	1nnnnnnnnnnnn0-u00n00-01-1-n	
82:	1111000000010111-11111un11	xnn-01000110nn-
10:	-0-101101111111111u01-1nu-u-1u	-1n0
11:	u1100-0-01001-01	xnuu
12:	-0001n1-nuu11-1u	u
13:	8nuuuu-uuun-	xnun
141	n110n01	XXX
15:	x-0011-01u11	x-nxx
16:	n	xxxn-
17:	1n1	- xn
18:	×1	×0
19:		
201		¥11
21:		***
22-		-xxx
23:		¥
2.4 -		
25:		
26:		*
27:		^x.
28:		· · · · · · · · · · · · · · · · · · ·
29.		
29:	×	¥¥¥
31.	A	*
31:	······	×
321	××	¥¥
331		××
	×	XXX
35:		X
36:	· · · · · · · · · · · · · · · · · · ·	××
	×-	XX
38:		
39:		×
40:		*
41:		×x-
42:	X-	×
431		
44:		×
45:		×
461		×
47:		
48:	× · · · · · · · · · · · · · · · · · · ·	X
491		
59:	×-	xx
51:		××-
521		
53:		¥
54:		*
12.12		
56:		
57:		
58:		
59:		
60.		
61:		· · · · · · · · · · · · · · · · · · ·
621	¥	
63:		
64-		·····×
651	¥	······································
661		······································
67.		
681	· · · · · · · · · · · · · · · · · · ·	······································
69 :	×	
701	××	××
	ss any key to continue	
Pres	as any key to concinue	

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-1 Collision attack on SHA-1

Conclusior

A better collision search algorithm

A less-naive search algorithm :

i = 1

repeat until a collision is found :

- pick a random message word M_i (backtrack sometimes)
- test if it follows the differential path for step *i*
 - if it does, then i = i + 1
- when *i* = 16, test if it follows the entire differential path

The very costly non-linear part is avoided (2) (2) (2), only the linear probabilistic part remains to be paid

-4:	00001111010010111000011111000011	
-31	010000001100100101010000111011000	
-21	011000101110101101110011111111010	
-1:	11101111110011011010101110001001	
00-	0110011101000101000000011000000001	n11u00000
00:	0110011101000101000000100000001	u9nu1
92:	0nnn11111101-11n-100 n00n0000n1-u101u	Genda
83-	-0-u111nnn-0100-01uu-	nu
	-0-u111nnn-0100-01uu- 0011nnn0010u1-011	xn0x
841	0011nnn001001-011	u-nx
05:	1-unuluuu0-000u01001unnn-n10	u-nx
86:	1n110n00000-10-000-0100n1un10110	nnnx0
07:	u1u01u11001-10-0n0111n1nn1-n-n	un11-00-
08:	1nnnnnnnnnnnn0-u00n00-01-1-n	
02:	1111000000010111-11111un11	xnn-010001
1.01	-0-101101111111111u01-1nu-u-1u	-10
11:		×041-
12:	-0001n1-nuul1-lu	A
131	8nuuuu-uuun-	*0
14-		
151		
161	n	x-n
17:	101	
18:	×1	×0
	×1	×n
19:20:		xu
		×u
21:	u	xxx
22:		- XX
23:		×
24:		
25:		
26:		¥
27:		
28:	······································	
29.		
30:		*
31:		¥
321	×-	
33:		×
34:	· · · · · · · · · · · · · · · · · · ·	
35:	*	¥
36-		· · · · · · · · · · · · · · · · · · ·
	×	XX
37:	××	XX
		××
39:		×
40:		×
41:		×
42:	x -	
43:		
441		×
45:		×
46:		×
471		
48:	×	
42:		
5.01	······	*
51:		×
52:		
531		*
54:		¥
55:		
561		
57:		
58-		
59: 60:		
60:		
	· · · · · · · · · · · · · · · · · · ·	
62:	×	
63:		
64:		
65:	×	
66:		
67:		
68:	××	
691	×	
70:	×	
Pre	ss any key to continue	

(二)、(四)、(三)、(三)、



Step

Collision attack on SHA-1 Chosen-Prefix

Chosen-Prefix Collision on SHA-1

Conclusion

Final collision search algorithm

Final collision search in 3 phases :

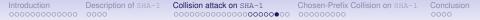
- Step 1 : handling the low-probability non-linear parts using the message block freedom
- Step 2 : apply the collision search speed-up techniques
- Step 3 : the remaining steps are verified probabilistically

Computation cost further reduced ☺ ☺ ☺ only steps ~22-80 have to be considered

- 4 -	00001111010010111000011111000011	
- 3.4	010000001100100101010000111011000	
-21	01100010111010101010101011111111010	
141	11101111111001101101010101110001001	
-1:	01100111010001010101010101000000001	n11u00000
		u0nu1
01:	0nnn111111101-11n-100	u0nu1
	n00n0000n1-u101u	nu
: 2.0	-0-u111nnn-0100-01uu-	
	0011nnn0010u1-011	xn0xu
	1-unu1uuu0-000u01001unnn-n10	u-nx
86:	1n110n00000-10-000-0100n1un10110	nnnx
87.1	u1u01u11001-10-0n0111n1nn1-n-n	un11-00-0
0.0 -	100000000000000-01-1-0	
- 00	1111000000010111-11111un11	xnn-01000110n
10.	-0-101101111111111u01-1mu-u-1u	100001-010001
	u1100-0-01001-01	×04
12:	-0001n1-nuu11-1u	Xu0
	-000111-10	YR
13:	n1	xnu
14:	n110n01 x-0	XXX
15:	x-0011-01u11	x-n
	n	XX
17:	1n1	- xn u
18:	×1	xn
19:		
201		¥11
:15		***
	×-	- **
1.00		
141		A
6741		
25:		
		×
27:		
28:	x -	×
29:		
301	×-	××
11:		X
	X-	
331		¥
24 -	×	
12.1		×
		×
21	×	×××
12	××	xxx
00.0		xx
91		*
: 64		×
:11		×
\$2:	×-	·····×
181		
14:		x
15:		¥
161		*
	××	
10.1	· · · · · · · · · · · · · · · · · · ·	*****
	×-	
149.1	×-	
:1:		×
53:		×
:4:		×
15 :		
6 :		
7:		
18.1		
10.1		
21		
142.1		
:14		
521	×	×-
184		
i45		
121	¥	¥
	x	
17.1		
58.1	·····	· · · · · · · · · · · · · · · · · · ·
201	× · · · · · · · · · · · · · · · · · · ·	×
191	××	×
re	ss any key to continue	

Step Step Step

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Collision search speed-up techniques

Several techniques to speed-up the collision search :

- message modifications [WYY05]
- neutral bits [BC04]
- boomerangs (or tunnels) [K06, JP07]

All these techniques trade message freedom degrees for a speed-up factor

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Chosen-Prefix Collision on SHA-DOOOOOOOO

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Collision attack on SHA-1

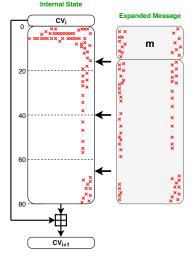
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- Prepare the collision search speed-up techniques by using the freedom degrees available

• Launch the collision search !



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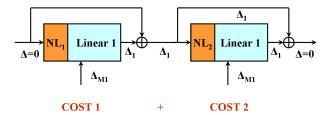
Chosen-Prefix Collision on SHA-1

Conclusion

Why chosen-prefix collision is hard for SHA-1

Can the SHA-1 collision attack be directly adapted for chosen-prefix scenario?

No: we can't remove the random difference on the chaining variable with the very small number of possible output differences of the linear path



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Why chosen-prefix collision is hard for SHA-1

Can the SHA-1 collision attack be directly adapted for chosen-prefix scenario ?

No : we can't remove the random difference on the chaining variable with the very small number of possible output differences of the linear path

We will use the recent SHA-1 collision attack [SBKAM17] as a black box. Denote $C (\simeq 2^{64.7})$ the computational cost for the last block.

Assume that we can use the same attack, for the same cost C, whatever is the input difference (this is possible thanks to the non-linear search algorithm). We validated this assumption in practice for several randomly chosen input differences.

ntroduction Description

Collision attack on SHA-1 Chose

Chosen-Prefix Collision on SHA-1

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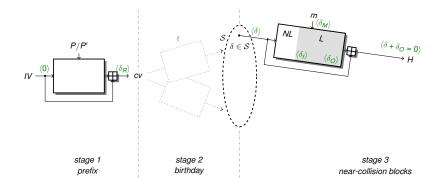
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Conclusion 0000

Birthday to the rescue!

Trick 1 : birthday search

Use **birthday search** to reduce the entropy of possible chaining variable differences



ntroduction Description of

Collision attack on SHA-1 Chos

Chosen-Prefix Collision on SHA-1

Conclusion

Birthday to the rescue!

Trick 1 : birthday search

Use **birthday search** to reduce the entropy of possible chaining variable differences

Assume that you have a set S of differences that you can reach on the output of the internal cipher, for a computation cost C.

phase 1 : compute the (random) state difference δ_R induced by the challenge prefix

phase 2 : apply birthday strategy to map difference δ_R to a difference δ that belongs to a certain set S (requires $\sqrt{\pi \cdot 2^n/|S|}$ computations)

phase 3 : apply the collision attack as explained previously (with a cost *C*) to map difference δ to a pair of colliding states

ntroduction Description of SHA

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Birthday to the rescue!

Trick 1 : birthday search

Use **birthday search** to reduce the entropy of possible chaining variable differences

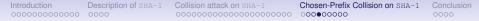
Stevens [S13] identified 192 possible output differences that can be reached for the same minimal cost C

```
phase 1 : O(1)
```

phase 2 : a birthday phase of 2^{77.1} computations

phase 3 : a collision phase of $C = 2^{64.7}$ computations

Total is 277.1 computations (birthday phase is dominating)



Relaxing the output differences constrains

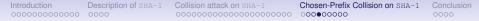
Trick 2a : generalized output differences

Using heuristics, we found **more allowable output differences** than previously known (576 instead of 192) for a cost *C*. This will increase S.

For a maximal computational cost of *C* per block, we found a set S of 576 elements (instead of 192)

70:		-		-	-	 	-		-	-	 	-	-	-	-	 	 -			-	-	-	-			-				-	-	 	 -		-			-		-	-	 -	-	-			-			-
71:	-			-	-	 	-	-	-		 	-	-	-		 	 -	-		-	-	-	-							-	-	 -	 -	-		-		-		-		 -		-	-		-			
72:	-				-	 			-	-	 	-	-	-	-	 	 -	-				-	-							-	-	 	 		-		-	-				 -		-			-	-		-
73:			-	-		 	-	-			 	-		-		 	 	-				-						-	-	-		 	 							-		 -		-	-		-			
74:		-		-	-	 			-	-	 	-	-	-	-	 	 -					-	-			-	-			-	-	 	 -	-	-	-		-				 		-			-		- 1	a.
75:				-	-	 	-		-		 	-	-			 	 	-		-	-	-		- 1	u	-			-	-	-	 	 -	-				-				 -		-	- 1	۱-	-			
76:	-		-	-	-	 	-	-	-	-	 	-	-	-	-	 	 	-	-	-	-	-	-			-		-	-	-	-	 	 -		-				-	-	-	 	-	-			-	-	- 3	ю
77:		-		-	-	 			-	-	 	-	-			 	 -	-			-	-	-				×	-		-	-	 	 -		-			-				 -					-	-	n-	-
78:					-	 	-	-			 	-	-	-		 	 				-	-	- 1	n,			×	-		-	-	 	 			-		-	-	-		 -		- 1	ц.		-		- 1	a
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Relaxing the output differences constrains

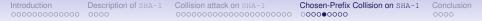
Trick 2b : generalized output differences

Accept **more costly differential paths** to further increase S (so that the birthday phase and collision phase have about the same computational cost).

For a maximal computational cost of $8 \cdot C$ per block, we found a set S of 8768 elements.

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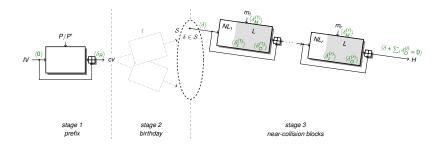
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Multiblocks : the more the merrier !

Trick 3 : multiblocks

Use **several blocks** to reach the collision after the birthday phase. It will increase the size of S and thus reduce the birthday phase cost.



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Multiblocks : the more the merrier !

Trick 3 : multiblocks

Use **several blocks** to reach the collision after the birthday phase. It will increase the size of S and thus reduce the birthday phase cost.

With a maximum cost of $3 \cdot C$, one obtains a set S containing about 2^{30} elements !

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The maximum length of the chain of blocks is 54 and the average is 17.

Multiblocks : the more the merrier !

Trick 3 : multiblocks

Use **several blocks** to reach the collision after the birthday phase. It will increase the size of S and thus reduce the birthday phase cost.

A three-phase attack :

phase 1 : compute the (random) state difference δ_R induced by the challenge prefix

phase 2 : apply birthday strategy to map difference δ_R to a difference δ that belongs to a certain set S

phase 3 : apply a few consecutive SHA-1 block attacks to slowly map difference δ to a pair of colliding states

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The clustering effect

Trick 4 : use the clustering effect

The attacker can sometimes **target several nodes simultaneously** to reduce the cost (because it is easier to hit one node out of many than a fixed one).

He will select **dynamically** the allowable differences at the output of each successive blocks. For that, we need a "map" of the whole situation, so we can decide which output difference I should be targeting at each new block. We will build a graph \mathcal{G} for that.

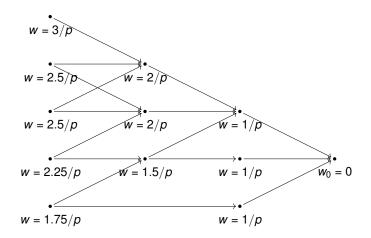
Introduction Description of SHA-

Collision attack on SHA-1

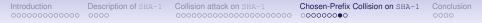
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Example of a graph G



(we assume that all the edges have probability p)



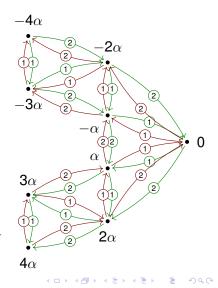
Building the graph \mathcal{G} and the set \mathcal{S}

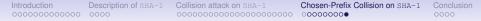
We call "bundle" a set of output differences that can be tried at the same time. We can build a graph \mathcal{G}' with the bundles.

Consider that you have :

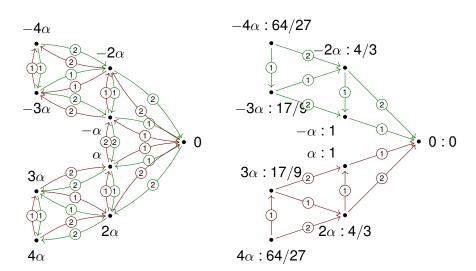
- a bundle {α, 2α} with costs 1 and 2 (green lines)
- a bundle {-α, -2α} with costs 1 and 2 (red lines)

The corresponding set S is $\{-4\alpha, -3\alpha, -2\alpha, -\alpha, 0, \alpha, 2\alpha, 3\alpha, 4\alpha\}$





Building the graph \mathcal{G} and the set \mathcal{S}



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Conclusion



Introduction

Description of SHA-1

Collision attack on SHA-1 Finding a linear differential path Finding a non-linear differential part Collision search speed-up techniques Launching the collision search

Chosen-Prefix Collision on SHA-1

Conclusion

Our results : SHA-1

We obtain a practical chosen-prefix collision attack on $\ensuremath{\mathtt{SHA-1}}$:

- with only 2^{66.9} computations (2^{69.4} with very pessimistic assumptions)
- only a small factor more costly than a classical collision attack
- all subparts of the attack verified experimentally

Function	Collision type	Complexity	Reference
SHA-1	free-start collision	2 ^{57.5}	[SKP16]
	collision	2 ⁶⁹ 2 ^{64.7}	[WYY05] [SBKAM17]
	chosen-prefix collision	2 ^{77.1} 2 ^{66.9}	[S13] our result

Our results : MD5

We can apply our strategy to MD5 (and others?) :

- less impressive because chosen-prefix collision attacks on MD5 are already rather cheap (thus the birthday phase becomes not competitive)
- however, when the number of blocks for the chosen-prefix collision is limited, our attacks becomes competitive

Function	Collision type	Complexity	Reference
MD5	collision	2 ⁴⁰ 2 ¹⁶	[WY05] [SSALMO09]
	chosen-prefix collision (9 blocks) (3 blocks) (1 block) (2 blocks)	2 ^{39.1} 2 ⁴⁹ 2 ^{53.2} 2 ^{46.5}	[SSALMO09] [SSALMO09] [SSALMO09] our result

Introduction Description

Collision attack on SHA-1

Chosen-Prefix Collision on SHA-000000000

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Conclusion

Thank you for your attention !

