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SESSION ID: CRYP-M01A

The Window Heuristic: Automating Differential Trail Search in ARX Ciphers with Partial Linearization Trade-offs

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Automated Cryptanalysis

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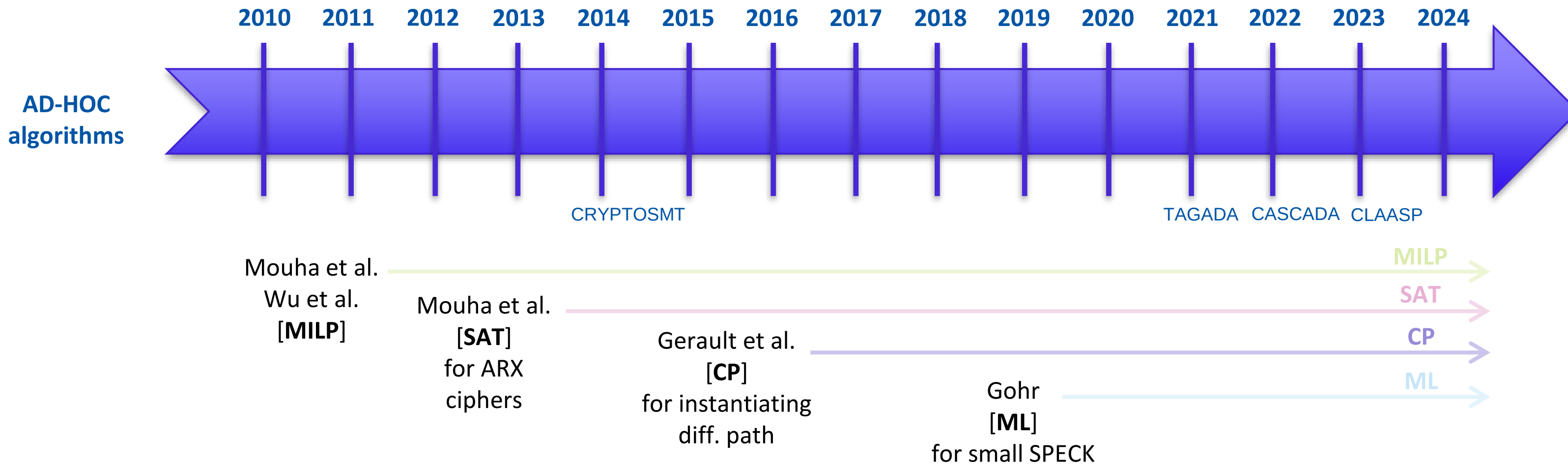
Timeline of Automated Cryptanalysis



Today almost all cryptanalysis are assisted by solvers (FSE 2024: two entire sessions just for pure automated cryptanalysis)

Automated cryptanalysis using declarative frameworks (SAT/MILP/CP/etc.) is generally slower or at best same as ad-hoc tools, but so much **more convenient**

Mainly on **differential** and **linear cryptanalysis**, but now also on integral distinguishers, cube attacks, meet-in-the-middle attacks, etc.

Solving time is a crucial aspect and can be impacted by:

- the framework you use (SAT/MILP/CP/etc.)
- the strategy of modeling
- the solver
- the type of problem studied / scale

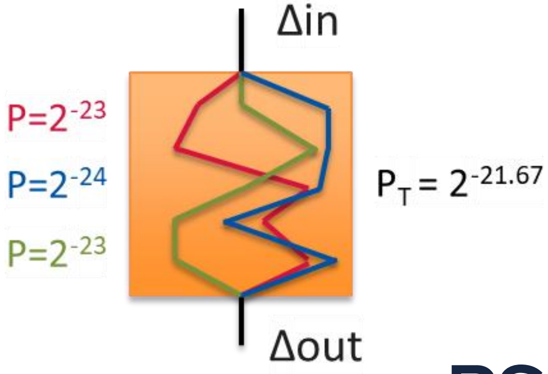
Automated Cryptanalysis for Differential Paths

Typically, for finding **differentials** or **differential trails**:

- Use **variables** to represent the various stages of the internal state bit differences during the round (and throughout the rounds)
- Use other variables to represent the **probability P** of the differential path (in -log2)
- Model a round of the cipher as a set of **declarative constraints** (Markov assumption !) to represent the difference propagation (either truncated or not). Use temporary variables if needed for certain components.
- Put all this into a system and use a **solver** on it.
- Can be combined with extra upper-level strategies (Matsui branch-and-bound, etc.)

One can:

- Find the best differential path / linear characteristic
- Enumerate the number of solutions
- Estimate the probability of a differential



Satisfiability Problem (SAT/SMT) for Cryptanalysis

All variables are **Boolean**, a constraint is a **Conjunctive Normal Form** (CNF – conjunction of disjunctions/clauses). Ex: $(a \vee b) \wedge (\neg a \vee c \vee d) \wedge (\neg b \vee c)$

- **Equality** is easy to model $x = y \Leftrightarrow (\neg x \vee y) \wedge (x \vee \neg y) = 1$
- **Linear layers** are easy to model, by combining the simple **XOR** model:
  $x = a \oplus b \Leftrightarrow (\neg a \vee b \vee x) \wedge (a \vee \neg b \vee x) \wedge (a \vee b \vee \neg x) \wedge (\neg a \vee \neg b \vee \neg x) = 1$
  (use dummy variables with combinations of XORs for multiple-inputs XORs)
- **Nonlinear layers (Sbox/AND/OR/Additions)** are more complex to model

Then, add a constraint to force the path to have a certain fixed probability P:

- If the solver returns SAT, we directly get a path with probability P.
- If the solver returns UNSAT, we run again this time with a probability P' < P (**drawback:** we need to iterate through decreasing target values).

SAT is very good **for all ciphers,** especially for **ARX ciphers**
SAT is very good for **finding optimal differential paths**, and good for **estimating the probability of a differential**.

Mixed Integer Linear Programming (MILP) for Cryptanalysis

All **variables** are **Boolean/integer/real**, a **constraint** is a linear **inequality**
Ex: $\{a + 4b + 2c \geq 3d + 7e\}$

- **Equality** is easy to model $x = y \Leftrightarrow \{x \geq y\}, \{y \geq x\}$
- **Linear layers** are easy to model, by using **4 inequalities**:
  $x = a \oplus b \Leftrightarrow \{a + b \geq x\}, \{a + x \geq b\}, \{b + x \geq a\}, \{a + b + x \leq 2\}$
  (use dummy variables with combinations of XORs for multiple-inputs XORs)
- **Nonlinear layers (Sbox/AND/OR/Additions)** are more complex to model

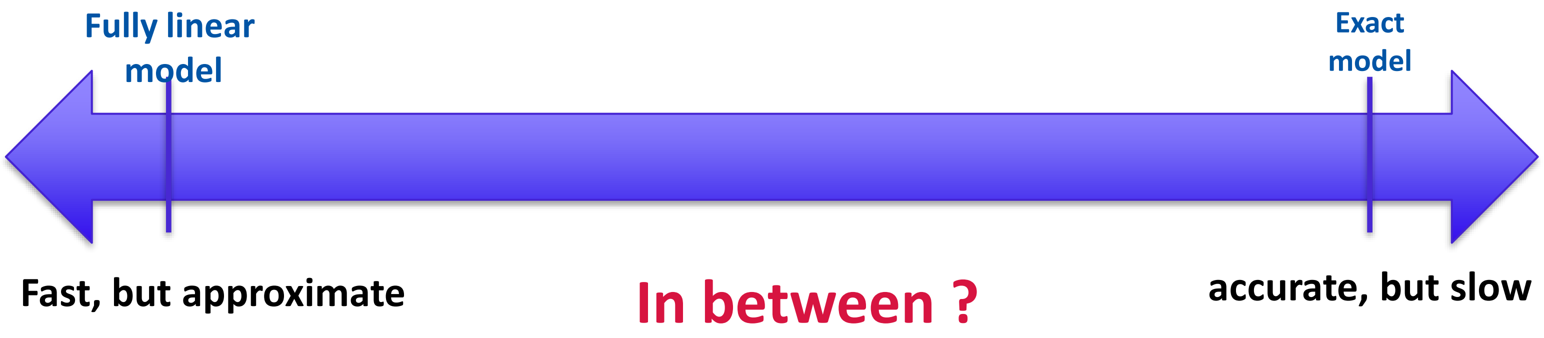
Then, add an **objective function** to minimize/maximize (encoding the path probability). Ex: $\{a + 3b + 2c + 5d\}$. The solver will directly return the best instance identified.

MILP is **good** on **most ciphers**.
MILP is **very good** for **finding optimal differential paths**, **less so** for **estimating the probability of a differential**.

The Problem with Modular Addition Modeling

- **Sboxes are relatively easy to model:** transform DDT into constraints
- **You can't use that strategy for ARX:** size is too large (32/64 bits)
- **To overcome this, researchers tried:**
  - to **fully linearise** mod. addition (Ex: MDx/SHA-x collision attacks): $x = a \boxplus b \approx a \oplus b$
  - to use Lipmaa-Moriai formula for XOR-differential propagation through addition:

    for $(\alpha, \beta \rightarrow \gamma)$: $eq(\alpha \ll 1, \beta \ll 1, \gamma \ll 1) \wedge (\alpha \oplus \beta \oplus \gamma \oplus (\beta \ll 1)) = 0$, with $eq(x, y, z) := (\neg x \oplus y) \wedge (\neg x \oplus z)$
  - to build the differential path iteratively, **only propagating the most likely differences** at each round (not representing the full cipher in a single model): partial DDT (pDDT)



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The Window Heuristic

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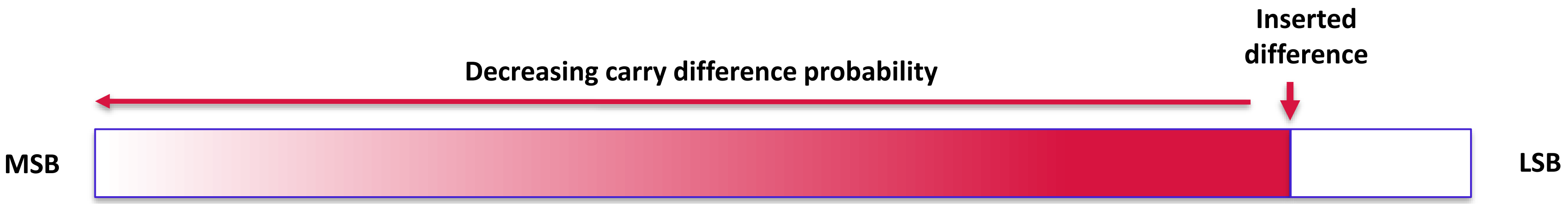
The Windows Heuristic Idea

A difference on bit i of a modular addition input can create a differential carry propagation from i to many later positions.

A linear model forces 0 carry difference propagation (but is fast), while exact model allows any propagation (but is slow).

- **Our observation:** good differential paths are (almost always) composed of modular addition diff. transitions that are propagating very little (if not at all), with only a few rounds with more propagation.



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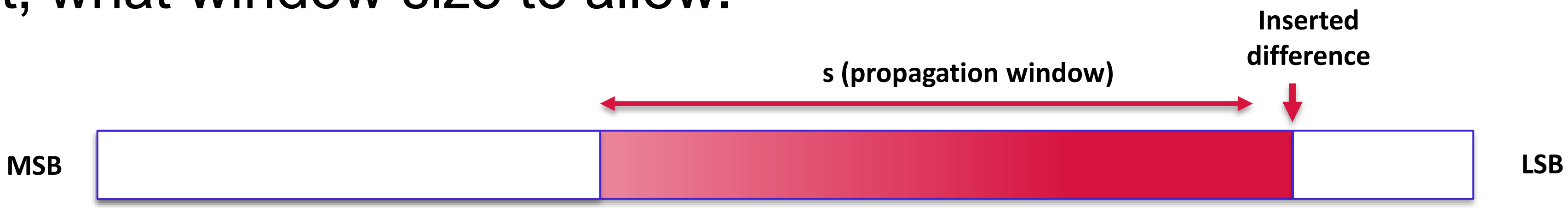
- **Our observation:** good differential paths are (almost always) composed of modular addition diff. transitions that are propagating very little (if not at all), with only a few rounds with more propagation.
- **Our idea:** generalize the linear and exact models, by allowing a difference **in the carry** to propagate a little bit (over s bits, aka the "**window**" of propagation). We can control for each modular addition and each difference bit of it, what window size to allow.



The Windows Heuristic Idea

Example:

Difference 0x080000001e4a0848

0000100000000000000000000000000000011110010010100000100001001000

+

Difference 0x08000000000e0808

0000100000000000000000000000000000000000000011100000100000001000

=

Difference 0x00000000f2400040

0000000000000000000000000000000011110010010000000000000001000000

Carry Difference 0x00000000ec040000

0000000000000000000000000000000011101100000001000000000000000000

window

Encoding the Window in MILP and SAT

- In **MILP**, for each n-bit modular addition with s-bit window, we add the (n - 1 - s) following new constraints on the corresponding carry C:

$$\sum_{i=1}^{s+1} C[j-i] \leq s, \qquad for\ all\ j \in [s+1, \dots, n-1].$$

- In **SAT**, for each n-bit modular addition with s-bit window, we add the $2^{2(s+1)}$ (n - 1 - s) following new constraints on the corresponding carry C:

$$\bigwedge_{j=s+1}^{n-1} \bigvee_{i=1}^{s+1} \neg(a[j-i] \oplus b[j-i] \oplus c[j-i])$$

The Heuristics

Window Heuristic does not create invalid paths, but potentially miss good ones (good to find attacks, bad to prove the non-existence of diff. paths).

We observed that **fixing s for an entire round** generally works well in practice, but this remains dependent on the cipher and attack we study.

Other possible heuristics:

- Force each round to contain few consecutive carry difference clusters (in general small for good differential paths)
- Restrict the **total** number of carry differences for each modular additions (in general not too large for good differential paths)

Unfortunately, these other heuristics did not improve the results.

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Results

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Application to SPECK Block Cipher

Version	Rounds	$\log_2(P_{DK})$	$\log_2(P_D)$	$\log_2(P_K)$	Ref.
SPECK-32/64	15	-94.0	-32.0	-62.0	[42]
	15	-85.0	-32.0	-53.0	[41]
	15	-73.0	-31.0	-42.0	$w_s = 0$
SPECK-48/96	14	-68.0	-43.0	-25.0	[42]
	14	-66.6	-43.0	-23.6	[41]
	14	-65.0	-34.0	-31.0	$w_s = 0$
	15	-89.0	-46.0	-43.0	[42]
	15	-83.5	-42.0	-41.5	[41]
	15	-75.0	-41.0	-34.0	$w_s = 0$
	16	-86.0	-43.0	-43.0	$w_s = 0$
SPECK-64/128	14	-88.0	-37.0	-51.0	[42]
	14	-72.0	-35.0	-37.0	[41]
	14	-66.0	-32.0	-34.0	$w_s = 0$
	15	-105.0	-45.0	-60.0	[42]
	15	-89.0	-42.0	-47.0	[41]
	15	-76.0	-33.0	-43.0	$w_s = 0$
	16	-103.0	-60.0	-43.0	[42]
	16	-85.0	-39.0	-46.0	$w_s = 0$
	17	-112.0	-62.0	-50.0	[42]
SPECK-128/256	16	-121.0	-76.0	-45.0	[42]
	16	-96.0	-52.0	-44.0	$w_s = 0$
	19	-190.0	-111.0	-79.0	[42]
	19	-171.0	-102.0	-69.0	$w_s = 0$

[41] Qin et al.
CoRR abs/2203.09741 - 2022

[42] Sadeghi et al.
Des. Codes Cryptography 2021

Application to ChaCha

Differential paths on ChaCha - comparison

Number of rounds	MILP + window heur.		SAT + window heuristic + "full window size"	MILP [7]	S-function [7]
	Pr	w_s	Pr	Pr	Pr
1	3	3	3	3	3
2	37	3	37	37	37
3	120	1	120	147	157
4	218	1	217	316	349

[7] Bellini et al. - IJACT 2023

Differential paths on ChaCha with MILP

r	Single-key			
	no-condition		conditions	
	Time	Pr	Time	Pr
1	2.17s	3	1.62s	3
2	13.2s	37	3.30s	37(0)
3	13.6s	147	319s	120(0)
4	36728s	316	48745s	218(1)

Boomerang Distinguishers on ChaCha

Number of rounds	Theoretical probability	Experimental probability
2	2^{-6} ($w_s = -1$, bottom part starting round 2)	$2^{-0.05}$
3	2^{-10} ($w_s = -1$, bottom part starting round 2)	$2^{-1.41}$
4	2^{-90} ($w_s = 0$, bottom part starting round 3)	-
5	2^{-154} ($w_s = 1$, bottom part starting round 3)	-
6	2^{-228} ($w_s = 1$, bottom part starting round 3)	-

Application to LEA Block Cipher

Differential paths on LEA

r	Window size		Others			
	Pr	w_s	Pr	Ref.	Pr	Ref.
12	107	9	112	[43]	107	[3]
13	123	1	134	[43]	127	[3]

Differential paths on LEA (time)

r	Single-key			
	no-condition		conditions	
	Time	Pr	Time	Pr
12	100709s	107/(-1)	73405s	107/(9)
13	609.53s	143/(-1)	7985s	123/(1)

[3] Bagherzadeh et al. - IET Information Sec. 2020

[43] Song et al. - ACISP 2016

Key recovery on LEA

Distinguisher			Key recovery		
Rounds	Pr	Ref.	Rounds	Complexity	Ref.
13	$2^{-123.79}$	[43]	14	$2^{124.79}$	[43]
13	$2^{-110.97}$	Ours ($w_s = 1$)	14	$2^{111.97}$	Ours ($w_s = 1$)

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Conclusion
-
Future Works

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Conclusion

- Maybe **better heuristics** exist with such window modeling trick
- Generalize the way we limit the carry, **maybe other good representations** are possible
- Generalize to **more complex operations** cases (multiplications, algebraic formula, ..)
- What about **linear cryptanalysis** ? Truncated differential cryptanalysis ?
- Incorporate the Window Heuristic into larger models with **key recovery**

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