

# **SPN-Hash: Improving the Provable Resistance Against Collision Attacks**



**JIALI CHOY, HUI HUI YAP, KHOONGMING KHOO,  
DSO NATIONAL LABORATORIES, SINGAPORE**

**JIAN GUO  
INSTITUTE FOR INFOCOMM RESEARCH, SINGAPORE**

**THOMAS PEYRIN, AXEL POSCHMANN  
NANYANG TECHNOLOGICAL UNIVERSITY,  
SINGAPORE**

**CHIK-HOW TAN  
TEMASEK LABORATORIES, NATIONAL UNIVERSITY  
OF SINGAPORE**

# What is SPN Hash?



- A Hash Function based on well-studied SPN structure.
- Generalized the optimal diffusion of SPN structure
  - So that more block sizes with good differential bounds can be constructed.
- First provable bound for true differential collision probability.
- Speed comparable to Grostl in software.
- Much lighter than SHA-3 candidates in hardware

# Motivation



<b>Hash</b>	<b>Proof of Security</b>	<b>Speed</b>
PKC-based , e.g. VSH (very smooth hash)	Collision can be reduced to solving hard problems	Very Slow

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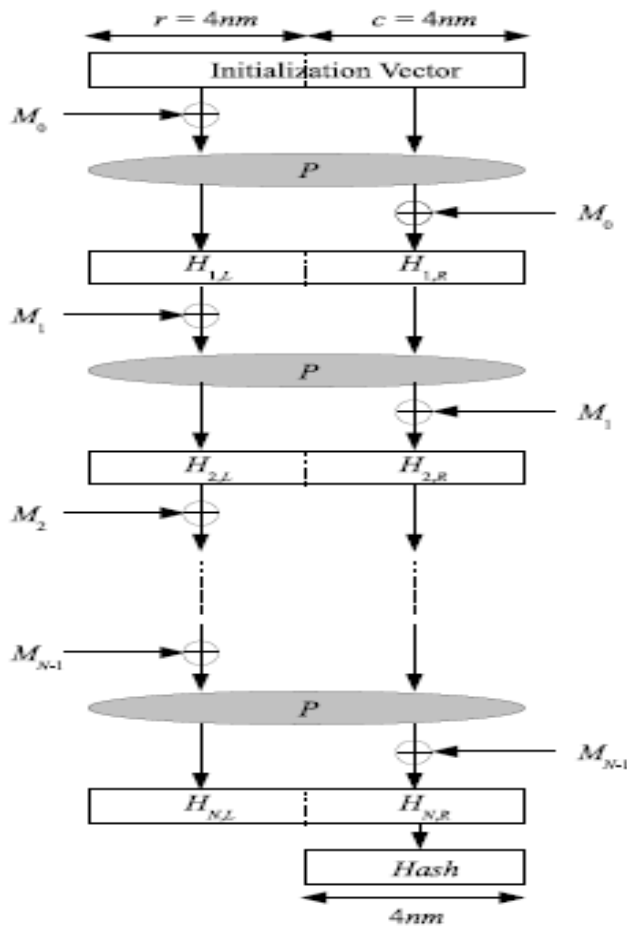
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SPN Hash		Low (true) DC, Provable collision resistance	Fast

# SPN Hash – Mode of Operation



1. Uses the JH mode of operation.
2. It is a sponge variant.
3. (a) **Sponge**:  $M_i$  only XORed to input.  
(b) **JH**:  $M_i$  is XORed to both input and output.
4. Reason for using JH:
  - (a) DC of  $P \Rightarrow$  collision resistance (similar to sponge).
  - (b) Pre-image resistant (similar to sponge).
  - (c)  $2^{\text{nd}}$  pre-image attack on sponge. No effective  $2^{\text{nd}}$  pre-image attack on JH.



# SPN Hash - Permutation P

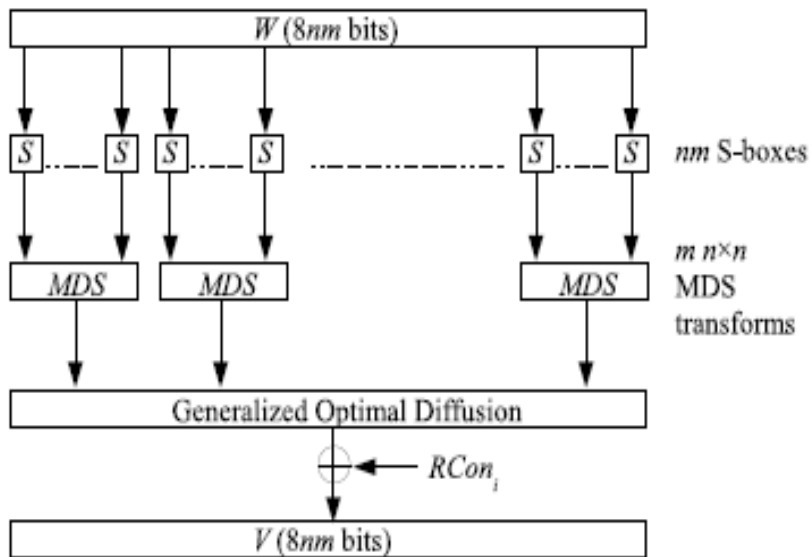


Fig. 4. The round function in permutation P

1.  $P$  iterates SPN structure **10 rounds**.
2. The substitution layer uses the AES S-box.
3. SPN similar to that used in AES:  
(a) There are  $m$  MDS's  
(b) Each MDS takes in  $n$  S-boxes
3. Known AES result,  $m=n$ .
4. In SPN hash,  $m$  divides  $n$ .

We design new component  
**Generalized Optimal Diffusion**  
to achieve non-square block size.

# SPN Hash - Permutation P



- **Q:** Why consider SPN design with non-square block size?  
**A:** So that we can design more block sizes.

<b>MDS Size (<math>n</math>)</b>	<b>Block Size (<math>n \times n</math>) AES-like SPN</b>	<b>Hash Size</b>
8 bytes	$8 \times 8 = 64$ bytes = 512 bit	256 bit
16 bytes	$16 \times 16 = 256$ bytes = 2048 bit	1024 bit

# SPN Hash - Permutation P



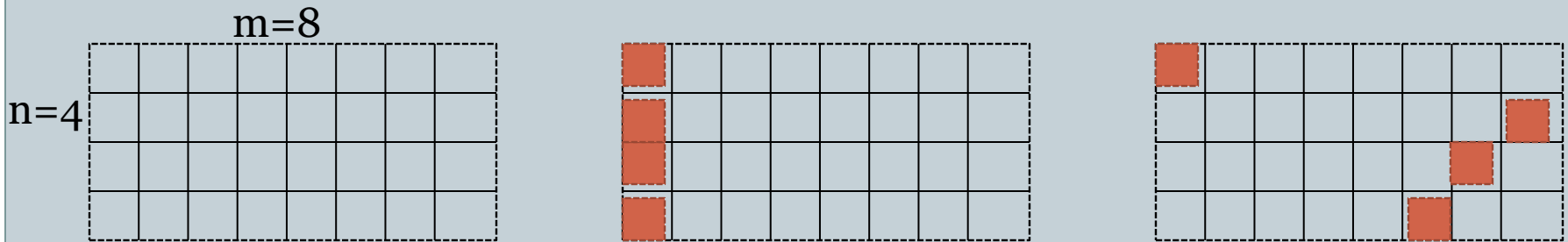
- Q:** Why consider SPN design with non-square block size?  
**A:** So that we can design more block sizes.

<b>MDS Size (<math>n</math>)</b>	<b>Block Size (<math>n \times n</math>) Square SPN</b>	<b>Hash Size</b>	<b>Block Size (<math>m \times n</math>), <math>m</math> divides <math>n</math> Our SPN</b>	<b>Hash Size</b>
8 bytes	$8 \times 8 = 64$ bytes $= 512$ bit	256 bit	$2 \times 8 = 16$ bytes= $128$ bit	64 bit
			$4 \times 8 = 32$ bytes= $256$ bit	128 bit
			$8 \times 8 = 64$ bytes= $512$ bit	256 bit
16 bytes	$16 \times 16 = 256$ bytes $= 2048$ bit	1024 bit	$2 \times 16 = 32$ bytes= $256$ bit	128 bit
			$4 \times 16 = 64$ bytes= $512$ bit	256 bit
			$8 \times 16 = 128$ bytes= $1024$ bit	512 bit
			$16 \times 16 = 256$ bytes= $2048$ bit	1024 bit

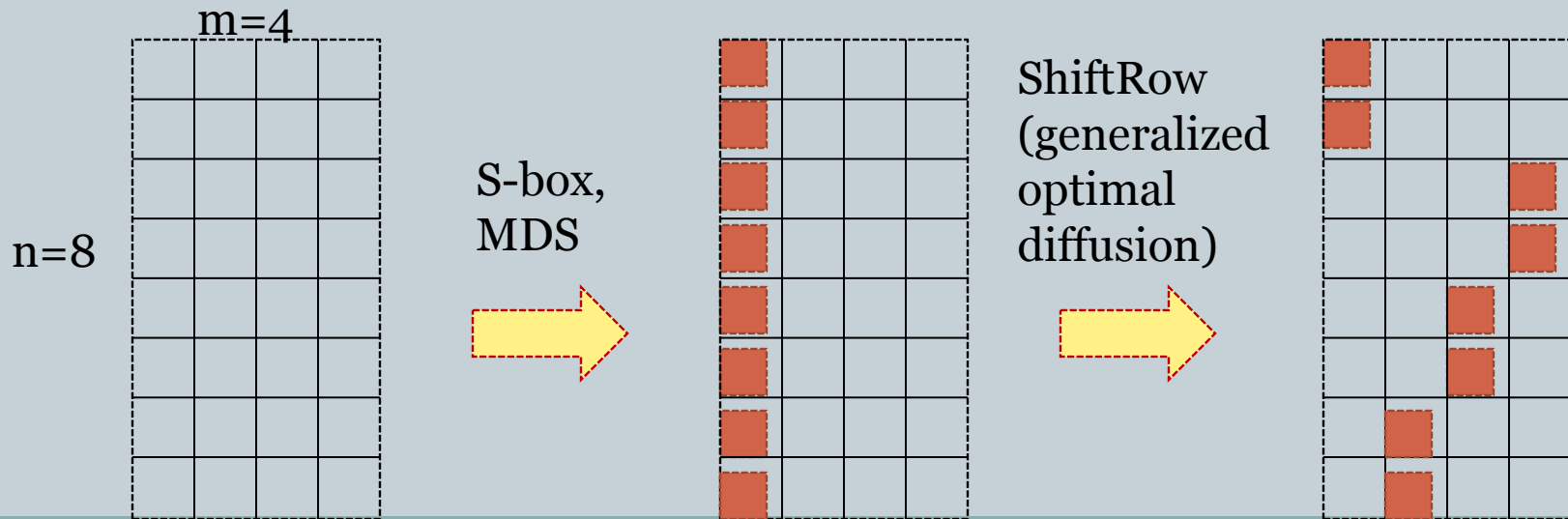
# How to Construct Non-Square Block Size



## Rijmen-Daemen's Construction (optimal diffusion)



## Our Construction (generalized optimal diffusion)



# Differential Results on our SPN

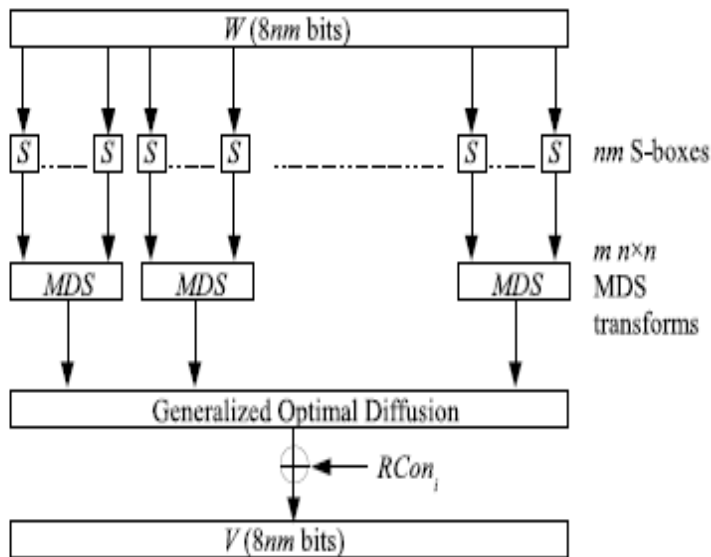


Fig. 4. The round function in permutation  $P$

- **Q:** Why care about differential probability?  
**A:** Collision  $\Leftrightarrow$  Zero Output Differential.
- **Rijmen-Daemen result:**  $m \geq n$ .  
Every 4 rounds  $\Leftrightarrow (n+1)^2$  active S-boxes
- **Our construction:**  $m$  divides  $n$ .  
Every 4 rounds  $\Leftrightarrow (m+1) \times (n+1)$  active S-boxes
- **Example:** Construct 32-byte block.
  - AES Result( $m=8, n=4$ ): **25 active S-box**.
  - Our Result ( $m=4, n=8$ ): **45 active S-box**

# Differential Results on our SPN



- Counting Active S-boxes  $\Leftrightarrow$  Characteristic Differential Probability (Uses Wide-trail strategy of Rijmen-Daemen in [IMA Conference on Crypto and Coding 2001, Springer LNCS 2260, pp.222])
- We want:  
True Differential Probability  $\Leftrightarrow$  Actual Collision Probability (Uses Park et al.'s SDS result [FSE 2003, Springer LNCS 2887, pp. 247])

# True Differential of SPN Hash



- **SPN Hash-128:** Block size = 256 bit. Hash output = 128-bit.
  - $n=8, m=4$  [32 AES S-box, Four  $8 \times 8$  MDS].
  - True differential probability (256-bit block)  $\leq 2^{-214.7}$ .
  - Differential collision probability  $\leq 2^{128} \times 2^{-214.7} = 2^{-86.7} < 2^{-64}$ .
- **SPN Hash-256:** Block size = 512 bit. Hash output = 256-bit.
  - $n=8, m=8$  [64 AES S-box, Eight  $8 \times 8$  MDS]
  - True differential probability (512-bit block)  $\leq 2^{-429.5}$ .
  - Differential collision probability  $\leq 2^{256} \times 2^{-429.5} = 2^{-173.5} < 2^{-128}$ .
- **SPN Hash-512:** Block size = 1024 bit. Hash output = 512-bit.
  - $n=16, m=8$  [128 AES S-box, Eight  $16 \times 16$  MDS].
  - True differential probability (1024-bit block)  $\leq 2^{-816}$ .
  - Differential collision probability  $\leq 2^{512} \times 2^{-816} = 2^{-304} < 2^{-256}$ .

# Comparison with Existing Hash



- Among SHA-2 and SHA-3 hashes, only one have true differential bound and that is ECHO.

	SPN Hash-512	ECHO-512
Block Size	1024 bit	2048 bit
True Diff Probability of Block	$2^{-816}$	$2^{-452}$
Output (after truncation)	512 bit	512 bit
True Differential Collision Probability (after truncation)	$2^{-304}$ (truncate 1024→512)	- (truncate 2048→512)

- True differential of ECHO block worse than SPN-hash block.
- ECHO truncate more bits, differential probability suffer even more.

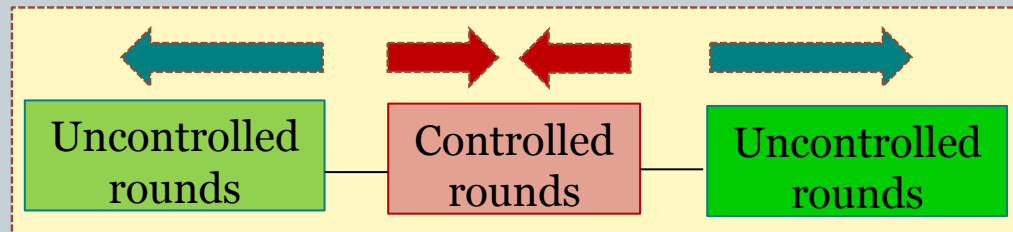
Why no collision prob for ECHO?



# Rebound Attack – Overview



- Divide an attack into two phases: Controlled rounds and Uncontrolled rounds



- Controlled rounds

- Efficient meet-in-the-middle
- Exploits available freedom degrees in the middle of a differential path
- Non Full Super-Sbox Analysis

- Uncontrolled rounds

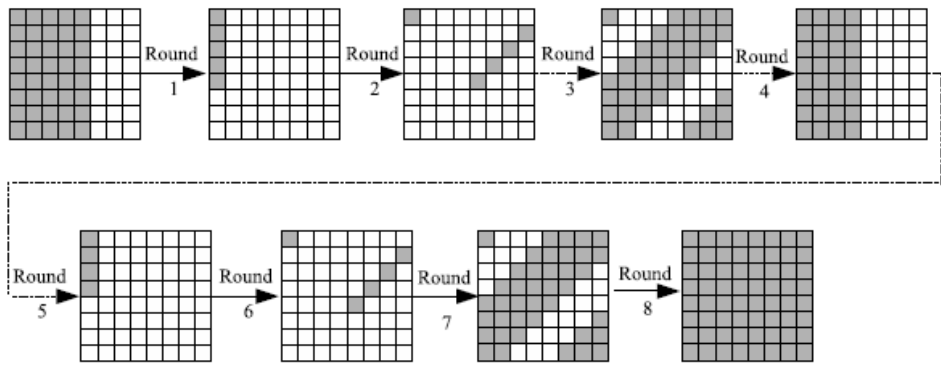
- Mainly probabilistic
- Solutions of the controlled rounds are computed backwards and forwards

- Can result in a **distinguishing attack**

# Rebound Attack

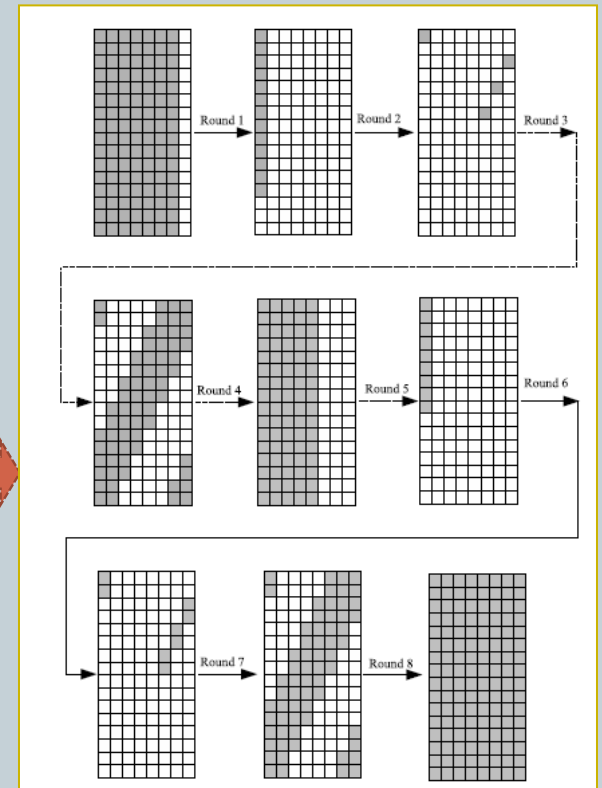


- View 512-bit and 1024-bit internal state of  $P$  as a  $8 \times 8$  and  $16 \times 8$  matrix of bytes
- 8-round differential paths
- **Coloured cell**: active byte;  
White cell: passive byte



512-bit  $P$

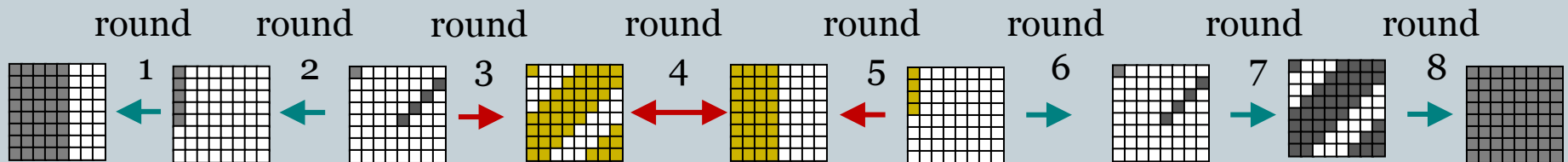
1024-bit  $P$



# Rebound Attack- Non-Full Active Super-Sbox



- The non-full active Super-Sbox method allows attacker to **control 3 rounds in the middle (controlled rounds)**: A starting point can be obtained with time 1 on average and  $2^8$  memory (512-bit  $P$ ) /  $2^{16}$  memory (1024-bit  $P$ )
- The rest of the path is fulfilled probabilistically (**uncontrolled rounds**): In the example of 512-bit  $P$  below, we have to pay a probability of approximately  $2^{-48}$
- Need to ensure **enough freedom degrees** to find a pair of values following the path: In example, need  $2^{48}$  starting points but can choose  $2^{72}$  differences at the start of controlled rounds



8-round differential path for 512-bit  $P$

# Rebound Attack



• **Q:** How does this translate to a distinguishing attack?

**A:** We obtained distinguishers:

- 512-bit  $P$ : Finding a valid pair for the whole 8-round path requires  $2^{48}$  operations and  $2^8$  memory. Ideal case requires  $2^{96}$  computations.
- 1024-bit  $P$ : Finding a valid pair for the whole 8-round path requires  $2^{88}$  operations and  $2^{16}$  memory. Ideal case requires  $2^{256}$  computations.

⇒ Secure against rebound attack since  $P$  comprises 10 round functions.

# Hardware Implementation



- Implement lightweight SPN Hash 128-bit and 256-bit.
- **Optimization:** Serialize the 8 by 8 MDS matrix over  $GF(2^8)$ .
- **Problem:** Not easy to find byte-based serialized 8 by 8 MDS matrix, by using method of PHOTON hash design.
- **Our Solution:** Use parallel copies of the PHOTON 8 by 8 MDS matrix over  $GF(2^4)$ .

# Serialized matrix over GF(2<sup>8</sup>)



$$Q = (A_{256})^8 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 4 & 2 & 11 & 2 & 8 & 5 & 6 \end{pmatrix}^8 = \begin{pmatrix} 2 & 4 & 2 & 11 & 2 & 8 & 5 & 6 \\ 12 & 9 & 8 & 13 & 7 & 7 & 5 & 2 \\ 4 & 4 & 13 & 13 & 9 & 4 & 13 & 9 \\ 1 & 6 & 5 & 1 & 12 & 13 & 15 & 14 \\ 15 & 12 & 9 & 13 & 14 & 5 & 14 & 13 \\ 9 & 14 & 5 & 15 & 4 & 12 & 9 & 6 \\ 12 & 2 & 2 & 10 & 3 & 1 & 1 & 14 \\ 15 & 1 & 13 & 10 & 5 & 10 & 2 & 3 \end{pmatrix}$$

$$X = (X_1 || X_2) \rightarrow (Q \cdot X_1 || Q \cdot X_2),$$

where  $X \in \text{GF}(2^8), X_1, X_2 \in \text{GF}(2^4)$

# Lightweight implementation



- Besides Serialized MDS, we also use other optimizations like compact AES S-box, efficient use of registers, etc...
- Comparison with SHA-3 candidates:

Digest size	Alg.	Ref.	Msg. size	Technology	Area [GE]	Latency [clk]	T'put@100KHz [kbps]	FOM [nbps/GE <sup>2</sup> ]
128	SPN-Hash-128		256	UMC 0.18	2777	710	36.1	2338
	SPN-Hash-128		256	<i>estimate</i>	4600	230	55.7	2627
256	SPN-Hash-256		512	UMC 0.18	4625	1430	35.8	837
	SPN-Hash-256		512	<i>estimate</i>	8500	230	111.3	1541
	BLAKE-32	[23]	512	UMC 0.18	13575	816	62.8	340
	GROSTL-224/256	[34]	512	AMS 0.35	14622	196	261.2	1222
	SKEIN-256-256	[34]	256	AMS 0.35	12890	1034	24.8	149

# Software Implementation



- Expect speed of SPN-Hash 256 comparable to Grostl-256 (22 cycles/byte).
  - Use same number of AES S-boxes.
  - T-Table implementation independent of MDS coefficients.
  - ShiftByte is done implicitly in T-table look-up.
  - SPN-Hash process 256-bit message in 10 rounds compared to Grostl-256 which process 512-bit message in 20 rounds.
- SPN-Hash 128 should run at similar speed.
  - Takes half the message bit, process half the operations.



# Conclusion



- We have designed new hash function SPN-hash.
  - Output Sizes: 128-bit, 256-bit, 512-bit
- Provable differential collision bound for all these sizes.
- Also secure against pre-image, 2<sup>nd</sup> pre-image and rebound attacks.
- Much lighter than existing SHA-3 candidates in Hardware.
- Efficiency comparable to Grostl in Software.

# Thank You!



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