SPN-Hash: Improving the Provable Resistance Against Collision Attacks

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What is SPN Hash?

- A Hash Function based on well-studied SPN structure.

- Generalized the optimal diffusion of SPN structure
  - So that more block sizes with good differential bounds can be constructed.

- First provable bound for true differential collision probability.

- Speed comparable to Grostl in software.

- Much lighter than SHA-3 candidates in hardware.
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<th>Hash</th>
<th>Proof of Security</th>
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<td>SPN Hash</td>
<td>Low (true) DC, Provable collision resistance</td>
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SPN Hash – Mode of Operation

1. Uses the JH mode of operation.
2. It is a sponge variant.
3. (a) **Sponge**: $M_i$ only XORed to input.
   (b) **JH**: $M_i$ is XORed to both input and output.
4. Reason for using JH:
   (a) DC of $P \Rightarrow$ collision resistance (similar to sponge).
   (b) Pre-image resistant (similar to sponge).
   (c) 2\textsuperscript{nd} pre-image attack on sponge. No effective 2\textsuperscript{nd} pre-image attack on JH.
1. \( P \) iterates SPN structure **10 rounds**.

2. The substitution layer uses the AES S-box.

3. SPN similar to that used in AES:
   (a) There are \( m \) MDS’s
   (b) Each MDS takes in \( n \) S-boxes

4. Known AES result, \( m=n \).

4. In SPN hash, \( m \) divides \( n \).

We design new component **Generalized Optimal Diffusion** to achieve non-square block size.
Q: Why consider SPN design with non-square block size?
A: So that we can design more block sizes.

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<th>Block Size (n×n) AES-like SPN</th>
<th>Hash Size</th>
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**A:** So that we can design more block sizes.

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<th>Hash Size</th>
<th>Block Size (m×n), m divides n Our SPN</th>
<th>Hash Size</th>
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<td>8 bytes</td>
<td>8×8 = 64 bytes = 512 bit</td>
<td>256 bit</td>
<td>2×8 = 16 bytes=128 bit</td>
<td>64 bit</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>4×8 = 32 bytes=256 bit</td>
<td>128 bit</td>
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How to Construct Non-Square Block Size Rijmen-Daemen’s Construction (optimal diffusion)

Rijmen-Daemen’s Construction (optimal diffusion)

m=8

n=4

Our Construction (generalized optimal diffusion)

m=4

n=8

S-box, MDS

ShiftRow (generalized optimal diffusion)
**Differential Results on our SPN**

- **Q:** Why care about differential probability?
  - **A:** Collision $\Leftrightarrow$ Zero Output Differential.

- **Rijmen-Daemen result:** $m \geq n$.
  - Every 4 rounds $\Leftrightarrow (n+1)^2$ active S-boxes

- **Our construction:** $m$ divides $n$.
  - Every 4 rounds $\Leftrightarrow (m+1) \times (n+1)$ active S-boxes

- **Example:** Construct 32-byte block.
  - AES Result $(m=8, n=4)$: **25 active S-box**.
  - Our Result $(m=4, n=8)$: **45 active S-box**
Differential Results on our SPN

- Counting Active S-boxes $\Leftrightarrow$ Characteristic Differential Probability (Uses Wide-trail strategy of Rijmen-Daemen in [IMA Conference on Crypto and Coding 2001, Springer LNCS 2260, pp.222])

- We want:
  True Differential Probability $\Leftrightarrow$ Actual Collision Probability (Uses Park et al.'s SDS result [FSE 2003, Springer LNCS 2887, pp. 247])
True Differential of SPN Hash

- **SPN Hash-128**: Block size = 256 bit. Hash output = 128-bit.
  - $n=8$, $m=4$ [32 AES S-box, Four $8 \times 8$ MDS].
  - True differential probability (256-bit block) $\leq 2^{-214.7}$.
  - Differential collision probability $\leq 2^{128} \times 2^{-214.7} = 2^{-86.7} < 2^{-64}$.

- **SPN Hash-256**: Block size = 512 bit. Hash output = 256-bit.
  - $n=8$, $m=8$ [64 AES S-box, Eight $8 \times 8$ MDS]
  - True differential probability (512-bit block) $\leq 2^{-429.5}$.
  - Differential collision probability $\leq 2^{256} \times 2^{-429.5} = 2^{-173.5} < 2^{-128}$.

- **SPN Hash-512**: Block size = 1024 bit. Hash output = 512-bit.
  - $n=16$, $m=8$ [128 AES S-box, Eight $16 \times 16$ MDS].
  - True differential probability (1024-bit block) $\leq 2^{-816}$.
  - Differential collision probability $\leq 2^{512} \times 2^{-816} = 2^{-304} < 2^{-256}$. 
Comparison with Existing Hash

- Among SHA-2 and SHA-3 hashes, only one have true differential bound and that is ECHO.

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<tr>
<th>Block Size</th>
<th>SPN Hash-512</th>
<th>ECHO-512</th>
</tr>
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<tr>
<td>True Diff Probability of Block</td>
<td>$2^{-816}$</td>
<td>$2^{-452}$</td>
</tr>
<tr>
<td>Output (after truncation)</td>
<td>512 bit</td>
<td>512 bit</td>
</tr>
<tr>
<td>True Differential Collision Probability (after truncation)</td>
<td>$2^{-304}$ (truncate 1024→512)</td>
<td>- (truncate 2048→512)</td>
</tr>
</tbody>
</table>

- True differential of ECHO block worse than SPN-hash block.
- ECHO truncate more bits, differential probability suffer even more.

Why no collision prob for ECHO?
Rebound Attack – Overview

- Divide an attack into two phases: Controlled rounds and Uncontrolled rounds

- Controlled rounds
  - Efficient meet-in-the-middle
  - Exploits available freedom degrees in the middle of a differential path
  - Non Full Super-Sbox Analysis

- Uncontrolled rounds
  - Mainly probabilistic
  - Solutions of the controlled rounds are computed backwards and forwards

- Can result in a **distinguishing attack**
Rebound Attack

- View 512-bit and 1024-bit internal state of $P$ as a $8 \times 8$ and $16 \times 8$ matrix of bytes
- 8-round differential paths
- Coloured cell: active byte;
  White cell: passive byte

512-bit $P$

1024-bit $P$
Rebound Attack- Non-Full Active Super-Sbox

- The non-full active Super-Sbox method allows attacker to control 3 rounds in the middle (controlled rounds): A starting point can be obtained with time 1 on average and $2^8$ memory (512-bit $P$) / $2^{16}$ memory (1024-bit $P$)

- The rest of the path is fulfilled probabilistically (uncontrolled rounds): In the example of 512-bit $P$ below, we have to pay a probability of approximately $2^{-48}$

- Need to ensure enough freedom degrees to find a pair of values following the path: In example, need $2^{48}$ starting points but can choose $2^{72}$ differences at the start of controlled rounds

8-round differential path for 512-bit $P
Rebound Attack

Q: How does this translate to a distinguishing attack?
A: We obtained distinguishers:

- 512-bit $P$: Finding a valid pair for the whole 8-round path requires $2^{48}$ operations and $2^8$ memory. Ideal case requires $2^{96}$ computations.

- 1024-bit $P$: Finding a valid pair for the whole 8-round path requires $2^{88}$ operations and $2^{16}$ memory. Ideal case requires $2^{256}$ computations.

⇒ Secure against rebound attack since $P$ comprises 10 round functions.
Hardware Implementation

- Implement lightweight SPN Hash 128-bit and 256-bit.

- **Optimization**: Serialize the 8 by 8 MDS matrix over GF(2^8).

- **Problem**: Not easy to find byte-based serialized 8 by 8 MDS matrix, by using method of PHOTON hash design.

- **Our Solution**: Use parallel copies of the PHOTON 8 by 8 MDS matrix over GF(2^4).
Serialized matrix over $\text{GF}(2^8)$

$$Q = (A_{256})^8 = \left( \begin{array}{c}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
2 & 4 & 2 & 1 & 1 & 2 & 8 & 5 & 6 \\
\end{array} \right)^8 = \left( \begin{array}{c}
2 & 4 & 2 & 1 & 1 & 2 & 8 & 5 & 6 \\
12 & 9 & 8 & 13 & 7 & 7 & 5 & 2 \\
4 & 4 & 13 & 13 & 9 & 4 & 13 & 9 \\
1 & 6 & 5 & 1 & 12 & 13 & 15 & 14 \\
15 & 12 & 9 & 13 & 14 & 5 & 14 & 13 \\
9 & 14 & 5 & 15 & 4 & 12 & 9 & 6 \\
12 & 2 & 2 & 10 & 3 & 1 & 1 & 14 \\
15 & 1 & 13 & 10 & 5 & 10 & 2 & 3 \\
\end{array} \right)$$

$$X = (X_1||X_2) \rightarrow (Q \cdot X_1|| Q \cdot X_2),$$

where $X \in \text{GF}(2^8), X_1, X_2 \in \text{GF}(2^4)$
Lightweight implementation

- Besides Serialized MDS, we also use other optimizations like compact AES S-box, efficient use of registers, etc...
- Comparison with SHA-3 candidates:

<table>
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<tr>
<th>Digest size</th>
<th>Alg.</th>
<th>Ref.</th>
<th>Msg. size</th>
<th>Technology</th>
<th>Area [GE]</th>
<th>Latency [clk]</th>
<th>T’put@100KHz [kbps]</th>
<th>FOM [nbps/GE²]</th>
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<tr>
<td>128</td>
<td>SPN-Hash-128</td>
<td>256</td>
<td>256</td>
<td>UMC 0.18</td>
<td>2777</td>
<td>710</td>
<td>36.1</td>
<td>2338</td>
</tr>
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<td></td>
<td>SPN-Hash-128</td>
<td>256</td>
<td>estimate</td>
<td></td>
<td>4600</td>
<td>230</td>
<td>55.7</td>
<td>2627</td>
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<tr>
<td>256</td>
<td>SPN-Hash-256</td>
<td>512</td>
<td>512</td>
<td>UMC 0.18</td>
<td>4625</td>
<td>1430</td>
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<td>8500</td>
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<td>111.3</td>
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<td>BLAKE-32</td>
<td>[23]</td>
<td>512</td>
<td>UMC 0.18</td>
<td>13575</td>
<td>816</td>
<td>62.8</td>
<td>340</td>
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<td>GROSTL-224/256</td>
<td>[34]</td>
<td>512</td>
<td>AMS 0.35</td>
<td>14622</td>
<td>196</td>
<td>261.2</td>
<td>1222</td>
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<td>12890</td>
<td>1034</td>
<td>24.8</td>
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Software Implementation

- Expect speed of SPN-Hash 256 comparable to Grostl-256 (22 cycles/byte).
  - Use same number of AES S-boxes.
  - T-Table implementation independent of MDS coefficients.
  - ShiftByte is done implicitly in T-table look-up.
  - SPN-Hash process 256-bit message in 10 rounds compared to Grostl-256 which process 512-bit message in 20 rounds.

- SPN-Hash 128 should run at similar speed.
  - Takes half the message bit, process half the operations.
Conclusion

- We have designed new hash function SPN-hash.
  - Output Sizes: 128-bit, 256-bit, 512-bit

- Provable differential collision bound for all these sizes.

- Also secure against pre-image, $2^{\text{nd}}$ pre-image and rebound attacks.

- Much lighter than existing SHA-3 candidates in Hardware.

- Efficiency comparable to Grostl in Software.
Thank You!

The full version of the paper can be found on ePrint 2012/234

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