Cryptanalysis of the 10-Round Hash and Full Compression Function of SHAvite-3-512

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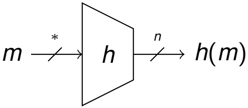
Africacrypt 2010
(initially discussed at ECRYPT2 Hash[3] workshop)

Outline

Overview

Cryptographic Hash Function



Hash function h maps arbitrary length input m to n-bit output $h(m)$

- Collision Resistance
  - find m, m' with $m \neq m'$ and $h(m) = h(m')$
  - birthday attack applies (freedom to choose $h(m)$)
  - generic complexity: $2^{n/2}$
- Second-Preimage Resistance
  - given $m, h(m)$ find m' with $m \neq m'$ and $h(m) = h(m')$
  - generic complexity: 2^n
- Preimage Resistance
  - given $h(m)$ find m
  - generic complexity: 2^n

Hash Function Cryptanalysis

- Recent improvements in hash functions cryptanalysis
  - last decade: major weaknesses in many hash functions
  - especially in MD-family of hash functions
  - NIST standard SHA-1 broken
- NIST SHA-3 competition [Nat07] (2008-2012)
  - find a successor of SHA-1 and SHA-2
  - similar as AES competition (2000)

SHA-3 Candidates

- 64 submissions to NIST call (October 2008)
- 51 round 1 candidates (December 2008)
  - many broken, too slow, not chosen, ...
- 14 round 2 candidates (August 2009)
  - chosen by NIST, tweaks allowed
- 5 finalists (fall 2010)
  - to focus analysis
- choose winner in 2011
  - standardize SHA-3 in 2012

How to Compare Attacks on SHA-3 Candidates?

Attacks on Building Blocks

- very different requirements for different designs
  - building blocks often not ideal
  - sponge: trivial “compression function” collisions/preimages
  - distinguishers on building blocks?
- when is an attack interesting?
  - NIST: not anticipated by the designers
  - if it extends to the hash function

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Attacks on Hash Function

- same requirements for all candidates
- a lot easier to compare
  - attacks on reduced hash function?
  - still hard to compare different security parameter(s)

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Attacks on Hash Function

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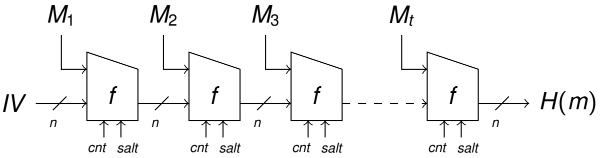
  - attacks on reduced hash function?
  - still hard to compare different security parameter(s)

Collection of SHA-3 Attacks:

http://ehash.iaik.tugraz.at/wiki/The_SHA-3_Zoo

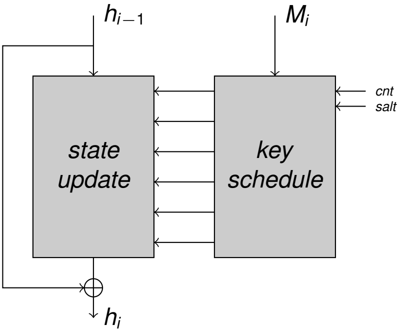
Overview

Description of SHAvite-3-512



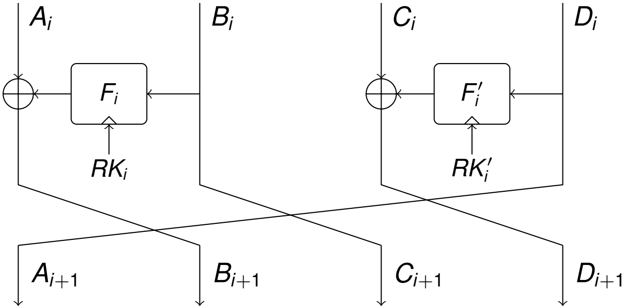
- Designed by Orr Dunkelman and Eli Biham [BD08]
  - Round 2 candidate
  - tweaked
- Iterated hash function
  - single-pipe construction
  - Haifa design principle

SHAvite-3-512 Compression Function



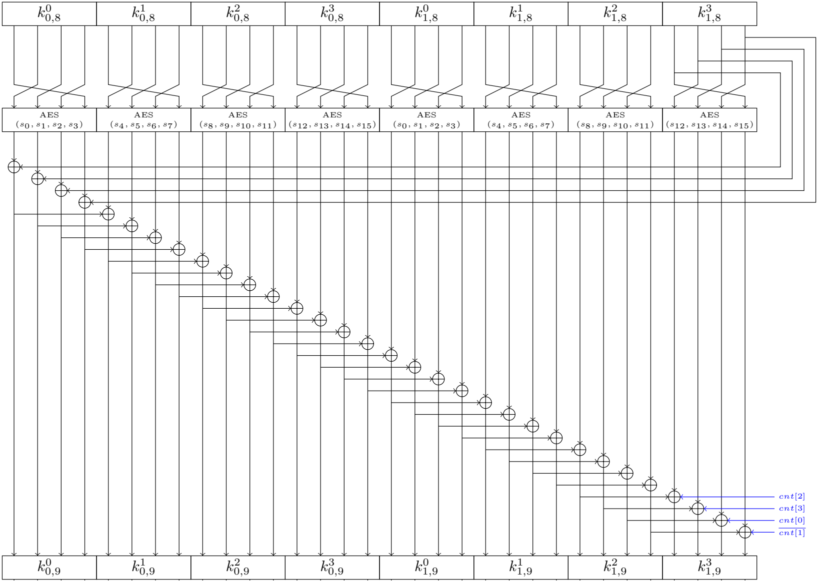
- block cipher in Davies-Meyer mode
- state update:
  - 14-round Feistel network (F-function: 4 AES rounds)
- key schedule:
  - parallel AES rounds with linear mixing layers

State Update

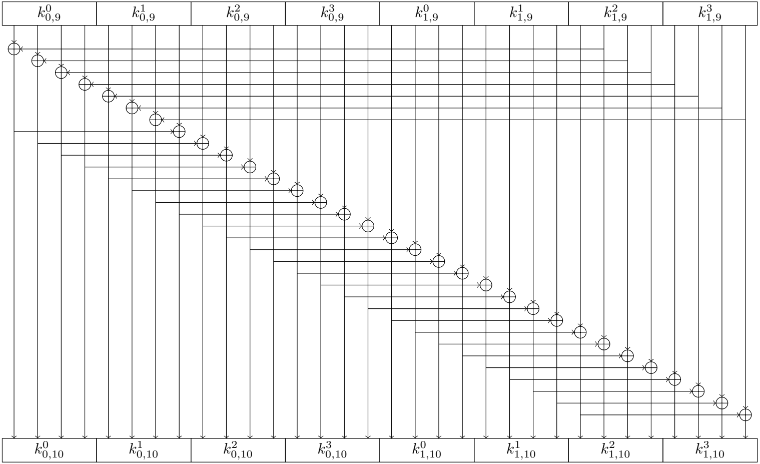


- $F_i(x) = AES(AES(AES(AES(x \oplus k^0_{0,i}) \oplus k^1_{0,i}) \oplus k^2_{0,i}) \oplus k^3_{0,i})$
- $AES(x) = MixColumns(ShiftRows(SubBytes(x)))$
- $RK_i = (k^0_{0,i}, k^1_{0,i}, k^2_{0,i}, k^3_{0,i})$

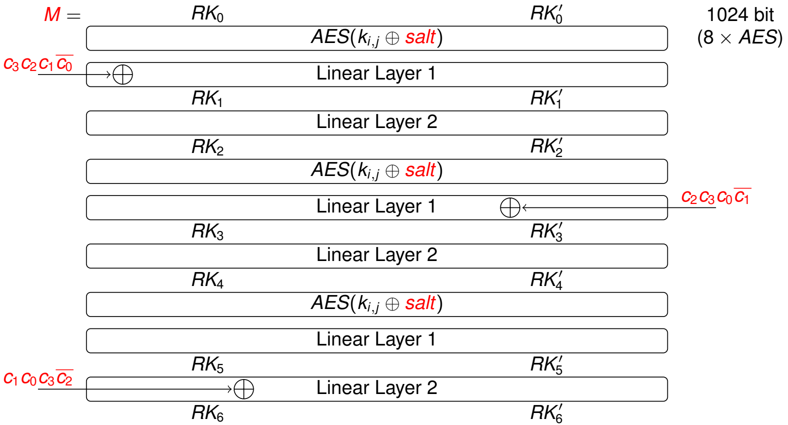
Key Schedule



Key Schedule



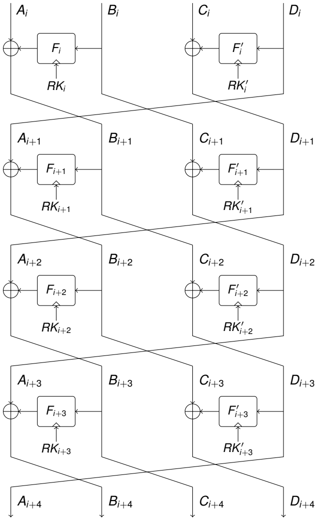
Key Schedule (schematic)



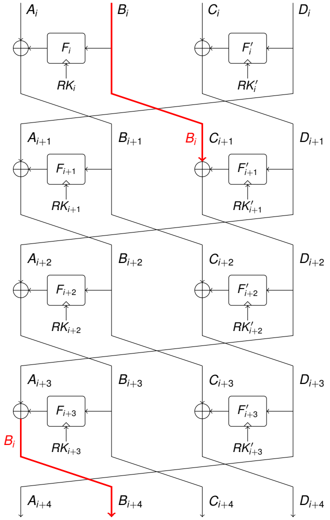
- Round 1: plain counter words added: $cnt = c_0 c_1 c_2 c_3$
- Round 2: inverted and shuffled counter words added

Overview

Cancellation Property [BDLF09]



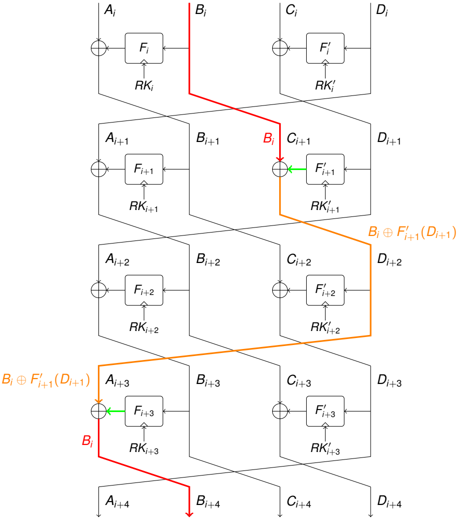
Cancellation Property [BDLF09]



- idea: keep B_i unchanged

$$B_{i+4} = B_i$$

Cancellation Property [BDLF09]



- idea: keep B_i unchanged

$$B_{i+4} = B_i$$

- when does this happen?

$$F_{i+3}(B_{i+3}) = F'_{i+1}(D_{i+1})$$

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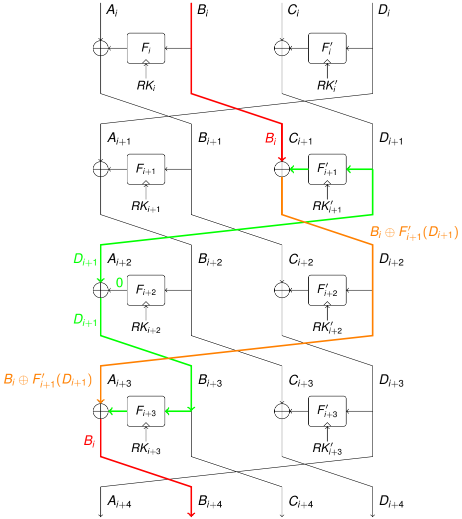


- idea: keep B_i unchanged

$$B_{i+4} = B_i$$

- when does this happen?

$$F_{i+3}(B_{i+3}) = F'_{i+1}(D_{i+1})$$

- or more specific:

$$F_{i+2}(B_{i+2}) = 0$$
$$RK_{i+3} = RK'_{i+1}$$

- second case:
  - two 128-bit conditions
  - but easier to fulfill
  - conditions can be “interleaved”

Interleaving

- interleave cancellation property with same value
  - $Z = B_i = B_{i+4}$
  - $Z = B_{i+2} = B_{i+4}$

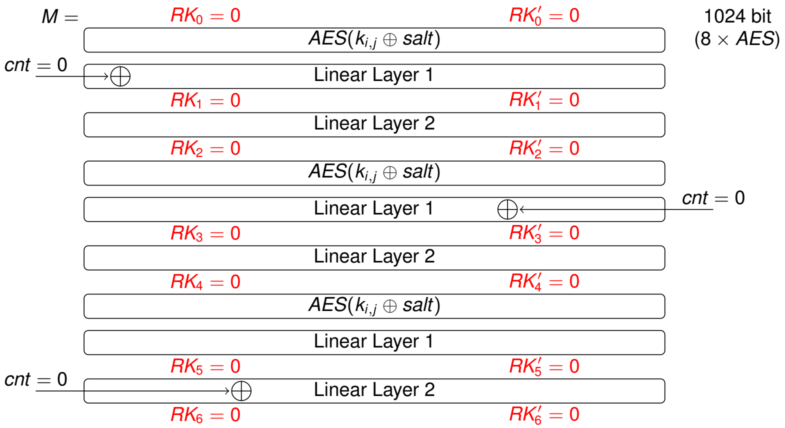
Interleaving

i	A_i	B_i	C_i	D_i	conditions
3	?	Z	?	?	
4	?	?	Z	D_4	
5	D_4	Z	?	$Z + F'_4(D_4)$	$F_5(Z) = 0$
6	$Z + F'_4(D_4)$	D_4	Z	D_6	$RK_6 = RK'_4$
7	D_6	Z	D_4	$Z + F'_6(D_6)$	$RK_7 = RK_5$
8	$Z + F'_6(D_6)$	D_6	Z	D_8	$RK_8 = RK'_6$
9	D_8	Z	D_6	$Z + F'_8(D_8)$	$RK_9 = RK_5$
10	$Z + F'_8(D_8)$	D_8	Z	D_{10}	$RK_{10} = RK'_8$
11	D_{10}	Z	D_8	$Z + F'_{10}(D_{10})$	$RK_{11} = RK_7$

- interleave cancellation property with same value
  - $Z = B_i = B_{i+4}$
  - $Z = B_{i+2} = B_{i+4}$
- conditions on state fulfill each other
  - we can choose $Z = F_5^{-1}(0)$

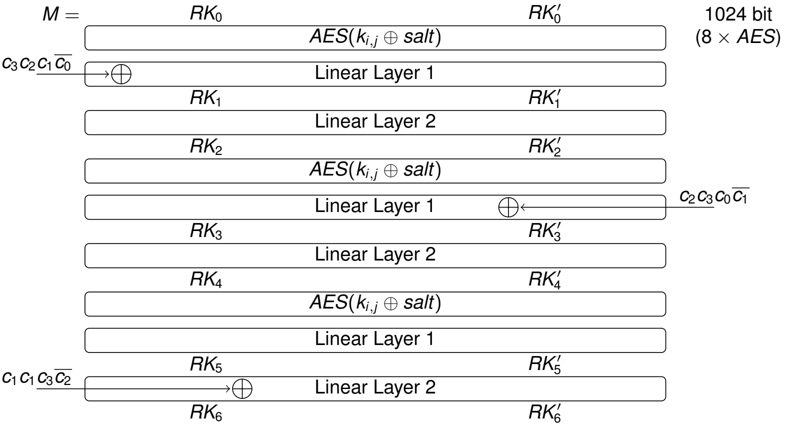
$\Rightarrow$ we get conditions only on keys (message expansion)

Weak Subkeys for SHAvite-3-512 (Round 1)



- construct all-zero subkeys [Pey09]
  - take the zero counter $cnt = 0$
  - choose salt such that $0 = AES(0 \oplus salt)$
  - $salt = 0x525252...52$

Weak Subkeys for SHAvite-3-512 (Round 2)



- tweak for SHAvite-3-512 (Round 2):
  - some counter words are inverted
  - all-zero subkey not possible anymore

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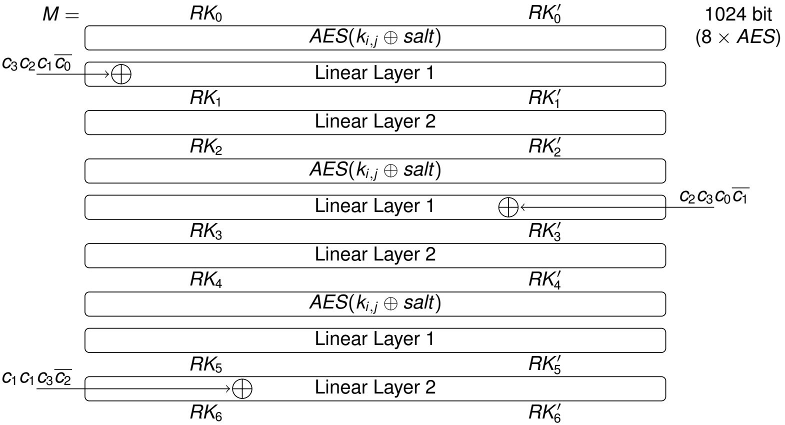


- tweak for SHAvite-3-512 (Round 2):
  - some counter words are inverted
  - all-zero subkey not possible anymore
- choose $c_2 c_3 c_0 \overline{c_1} = 0$ (valid counter!)
  - many round keys get zero

Weak Subkeys for SHAvite-3-512 (Round 2)

i	RK_i $k^0_{0,i}$	$k^1_{0,i}$	$k^2_{0,i}$	$k^3_{0,i}$	RK'_i $k^0_{1,i}$	$k^1_{1,i}$	$k^2_{1,i}$	$k^3_{1,i}$	r
0	?	?	?	?	?	?	?	?	M
1	?*	?	?	?	?	?	?	0	1
2	0	?	?	?	?	0	0	0	
3	0	?	?	?	0	0	0	0	2
4	0	?	0	0	0	0	0	0	
5	0	0*	0	0	0	0	0	0	3
6	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	4
8	0	0	0	0	0	0	0	0	
9	0	0	0	0*	0	0	0	0	5
10	0	0	0	0	0	0	0	0	
11	0	0	0	0	0	0	0	0	6
12	0	0	0	0	0	0	0	0	
13	0	0	0	0	0	0	?*	?	7

- key conditions are fulfilled for $Z = B_3 = B_5 = \cdots = B_{13}$

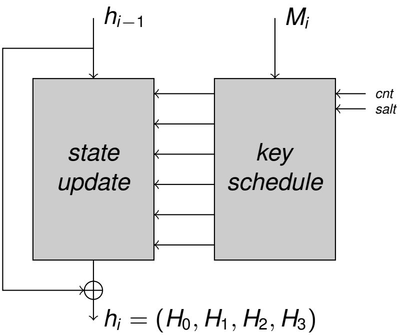
Weak Subkeys for SHAvite-3-512 (Round 2)

i	RK_i				RK'_i				r
	$k^0_{0,i}$	$k^1_{0,i}$	$k^2_{0,i}$	$k^3_{0,i}$	$k^0_{1,i}$	$k^1_{1,i}$	$k^2_{1,i}$	$k^3_{1,i}$	
0	?	?	?	?	?	?	?	?	M
1	?*	?	?	?	?	?	?	0	1
2	0	?	?	?	?	0	0	0	
3	0	?	?	?	0	0	0	0	2
4	0	?	0	0	0	0	0	0	
5	0	0*	0	0	0	0	0	0	3
6	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	4
8	0	0	0	0	0	0	0	0	
9	0	0	0	0*	0	0	0	0	5
10	0	0	0	0	0	0	0	0	
11	0	0	0	0	0	0	0	0	6
12	0	0	0	0	0	0	0	0	
13	0	0	0	0	0	0	?*	?	7

- key conditions are fulfilled for $Z = B_3 = B_5 = \cdots = B_{13}$
- in fact we can find 2^{224} weak salts

Overview

Compression Function



1. partial preimage attack (of 128 bits):
   - given H_2, compute $M_i, cnt, salt, h_i$
2. collision or preimage only on H_0, H_1, H_3
   - complexity 2^{192} and 2^{384} ($|h_i| = 512$)

The 14-Round Characteristic

i	A_i	B_i	C_i	D_i	conditions
0	?	?	?	?	
1	?	?	?	?	
2	?	X	?	?	
3	?	Z	X	?	
4	?	Y	Z	D_4	
5	D_4	Z	Y	$Z + F'_4(D_4)$	$F_5(Z) = 0$
6	$Z + F'_4(D_4)$	D_4	Z	D_6	$RK_6 = RK'_4$
7	D_6	Z	D_4	$Z + F'_6(D_6)$	$RK_7 = RK_5$
8	$Z + F'_6(D_6)$	D_6	Z	D_8	$RK_8 = RK'_6$
9	D_8	Z	D_6	$Z + F'_8(D_8)$	$RK_9 = RK_7$
10	$Z + F'_8(D_8)$	D_8	Z	D_{10}	$RK_{10} = RK'_8$
11	D_{10}	Z	D_8	$Z + F'_{10}(D_{10})$	$RK_{11} = RK_9$
12	$Z + F'_{10}(D_{10})$	D_{10}	Z	?	$RK_{12} = RK'_{10}$
13	?	Z	D_{10}	?	$RK_{13} = RK_{11}$
14	?	?	Z	?	

- choose (M, cnt, $salt$) according to key conditions
- compute Z in round 5
- we know that we get $C_{14} = Z$
- missing: compute X, Y for given $H_2 = C_0 \oplus C_{14}$

Partial Preimage for 14-Rounds

i	A_i	B_i	C_i	D_i	conditions
0	?	?	?	?	
1	?	?	?	?	
2	?	X	?	?	
3	?	Z	X	?	
4	?	Y	Z	D_4	
5	D_4	Z	Y	$Z + F'_4(D_4)$	$F_5(Z) = 0$
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12	$Z + F'_{10}(D_{10})$	D_{10}	Z	?	$RK_{12} = RK'_{10}$
13	?	Z	D_{10}	?	$RK_{13} = RK_{11}$
14	?	?	Z	?	

- write $H_2 = C_0 \oplus C_{14}$ as a function of X, Y, Z:

$$H_2 = F_2(X) + F'_0(X + F_1(Z + F_4(Y) + F'_2(Y + F_3(Z))))$$

Partial Preimage for 14-Rounds

i	A_i	B_i	C_i	D_i	conditions
0	?	?	?	?	
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12	$Z + F'_{10}(D_{10})$	D_{10}	Z	?	$RK_{12} = RK'_{10}$
13	?	Z	D_{10}	?	$RK_{13} = RK_{11}$
14	?	?	Z	?	

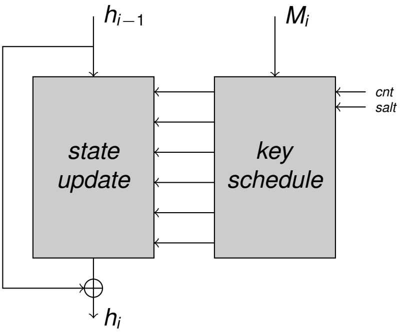
- write $H_2 = C_0 \oplus C_{14}$ as a function of X, Y, Z:

$$H_2 = F_2(X) + F'_0(X + F_1(Z + F_4(Y) + F'_2(Y + F_3(Z))))$$

- solve for X, Y using birthday effect (2^{64}):

$$F'^{-1}_0(H_2 + F_2(X)) + X = F_1(Z + F_4(Y) + F'_2(Y + F_3(Z)))$$

Results for the Full Compression Function

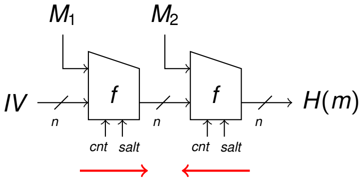


- collision attack:
  - complexity 2^{192} and 2^{128} memory
- preimage attacks:
  - complexity 2^{384} and 2^{128} memory
  - complexity 2^{448} without memory

with chosen salt and chosen counter

Overview

Hash Function



Can we extend the attack on the compression function to an attack on the hash function?

- in general: yes
  - if the design is single-pipe,
  - and we fix one output word:
  - do a meet-in-the-middle attack on 512 bit chaining value (two blocks needed)
- in this case: no
  - because salt is different for each 2nd block

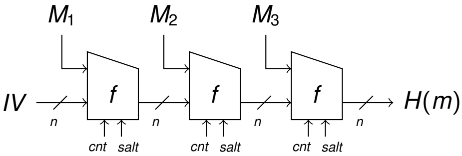
$\Rightarrow$ extend the attack of [BDLF10] by one round

Characteristic for 10 Rounds

i	A_i	B_i	C_i	D_i	condition
0	?	?	?	?	
1	?	?	?	?	
2	?	X	?	?	
3	?	Z_7	X	?	
4	?	Y	Z_7	D_4	
5	D_4	Z_5	Y	$Z_7 + F'_4(D_4)$	
6	$Z_7 + F'_4(D_4)$	$D_4 + F_5(Z_5)$	Z_5	D_6	$F_6(D_4 + F_5(Z_5)) = F'_4(D_4)$
7	D_6	Z_7	?	$Z_5 + F'_6(D_6)$	
8	$Z_5 + F'_6(D_6)$	$D_6 + F_7(Z_7)$	Z_7	?	$F_8(D_6 + F_7(Z_7)) = F'_6(D_6)$
9	?	Z_5	?	?	
10	?	?	Z_5	?	

- fulfill conditions by carefully choosing subkey values [BDLF10]:
  $$(k^1_{0,4}, k^2_{0,4}, k^3_{0,4}) = (k^1_{1,6}, k^2_{1,6}, k^3_{1,6}) \text{ and } k^0_{0,4} + k^0_{1,6} = F_5(Z_5)$$
  $$(k^1_{0,6}, k^2_{0,6}, k^3_{0,6}) = (k^1_{1,8}, k^2_{1,8}, k^3_{1,8}) \text{ and } k^0_{0,6} + k^0_{1,8} = F_7(Z_7)$$
- compute $H_2 = C_0 \oplus C_{14}$ as a function of X, Y, Z_5, Z_7
  - using birthday effect again (2^{64})

Hash Function Attack on 10 Rounds



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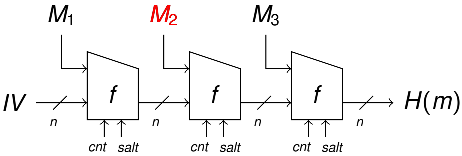
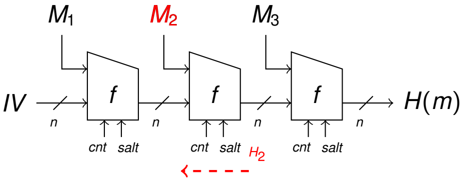


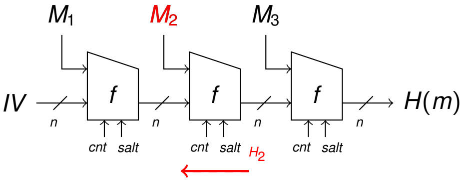
1. find a message (M_2) according to key conditions (2^{224})

Hash Function Attack on 10 Rounds



1. find a message (M_2) according to key conditions (2^{224})
2. find all 2^{128} partial preimages (solutions for X, Y)
   - using cycle finding algorithm
   - total complexity: $2^{128+64} = 2^{192}$

Hash Function Attack on 10 Rounds



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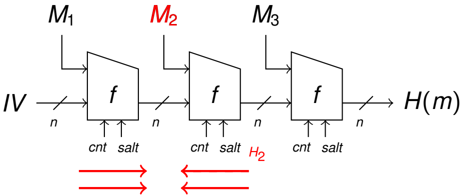
3. to find a preimage for the compression function
   - repeat previous steps 2^{256} times
   - total complexity: $2^{224+256} = 2^{480}$

Hash Function Attack on 10 Rounds



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   - using cycle finding algorithm
   - total complexity: $2^{128+64} = 2^{192}$
3. to find a preimage for the compression function
   - repeat previous steps 2^{256} times
   - total complexity: $2^{224+256} = 2^{480}$
4. construct a second-preimage for the hash function
   - using unbalanced meet-in-the-middle attack
   - complexity: 2^{497} and 2^{16} memory

Overview

Conclusion

- Attacks on SHAvite-3-512:
  - Full Compression Function
  - 10/14 Rounds for the Hash Function
- Why does it work?
  - *salt*, *cnt* inputs: weaker compression functions (harder to extend attacks to hash function)
  - regular key schedule
  - Feistel: we can keep properties for many rounds
  - single-pipe design
- Security margin already rather small
- Attack did not use properties of AES yet

  (even works for ideal permutation instead of AES rounds)

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