The Usage of Counter Revisited: Second-Preimage Attack on New Russian Standardized Hash Function

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joint work with:

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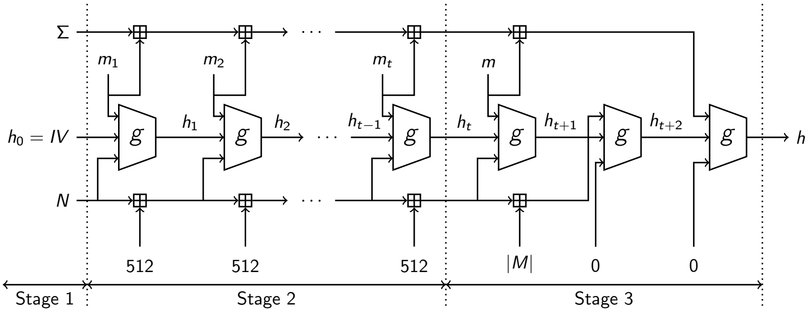
Streebog: new Russian hash function.

- New hash function standard in Russia.
- Standardized name: GOST R 34.11-2012
- Nickname of that function: Streebog.
- Previous standard: GOST R 34.11-94.
  - Theoretical weaknesses.
  - Rely on the GOST block cipher from the same standard.
  - This block cipher has also been weakened by third-party cryptanalysis.

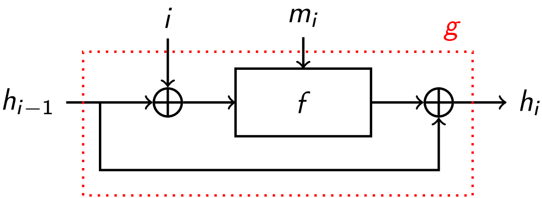
Specifications: domain extension.

- Two versions: `Streebog-256` and `Streebog-512`.
- 10^* padding: $m_1||\cdots||m_t||m$ (blocks of 512 bits).
- Compression function: g.
- Checksum: Σ, over the message blocks m_i (addition modulo 2^{512}).
- Counter: N, HAIFA input to g over the number of processed bits.
- Three stages: initialization, message processing and finalization.



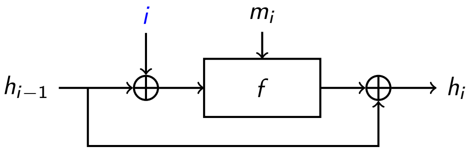
Specifications: compression function.

- Simplification: the counter counts #blocks, not #bits.
- g compresses (h_{i-1}, i, m_i) to h_i using: $h_i = f(h_{i-1} \oplus i, m_i) \oplus h_{i-1}$.
- Our attack is independent of the specifications of f (deterministic).



- g is one instantiation of a HAIFA compression function.
- The counter is simply XORed to the input of the f function.

Equivalent compression function.



$$h_i = h_{i-1} \oplus f(h_{i-1} \oplus i, m_i) \quad \Longleftrightarrow$$

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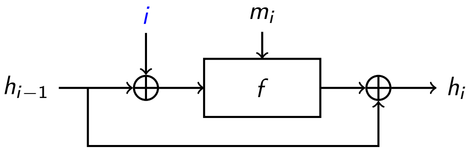

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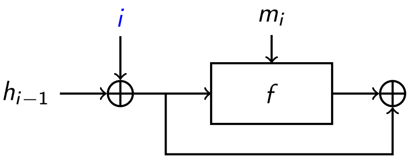

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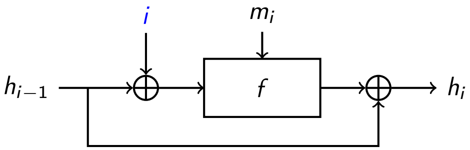

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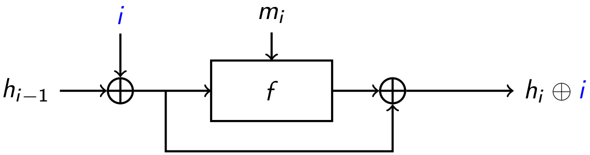

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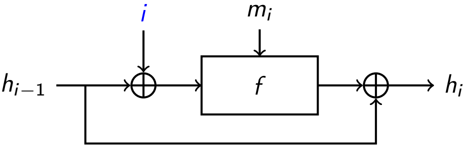

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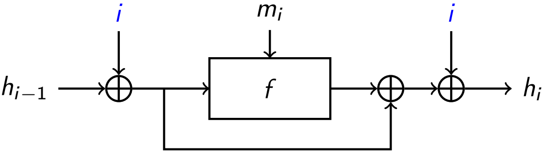

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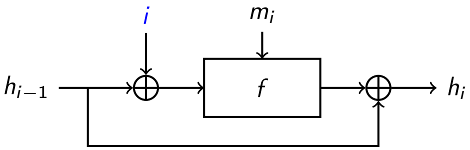
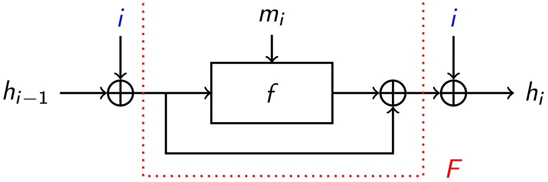


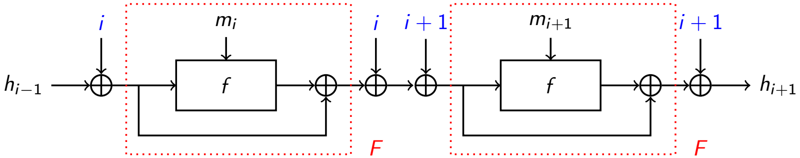
$$h_i = h_{i-1} \oplus f(h_{i-1} \oplus i, m_i) \iff \begin{cases} h_i = F(h_{i-1} \oplus i, m_i) \oplus i, \\ F(x, m_i) = f(x, m_i) \oplus x. \end{cases}$$



The function F is independent of the counter value!

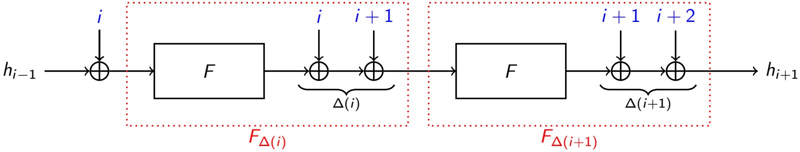
Iteration of the equivalent compression function.

- We have an equivalent representation of the compression function.
- Its iteration allows to combine the counter additions.



$$\Delta(i) \stackrel{\text{def}}{=} i \oplus (i+1),$$

$$F_{\Delta(i)}(X, Y) \stackrel{\text{def}}{=} F(X, Y) \oplus \Delta(i).$$



Relations between functions $F_{\Delta(i)}$ for $1 \leq i \leq t$ (1/2).

Recall that t is the number of full blocks $m_1 || \cdots || m_t || m$, $|m| < 512$.
We observe that:

- For all even i, $\Delta(i) = i \oplus (i+1) = 1$.
  $\Longrightarrow$ The same function F_1 is used every other time.
- Sequence of $\Delta(i)$ is very structured.

i:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
$\Delta(i)$:	1	3	1	7	1	3	1	15	1	3	1	7	1	3	1	31	1	3	1	7	1	3	1	15

Let $s > 0$, and denoting $\langle \mathtt{i} \rangle$ the s-bit binary representation of $i < 2^s - 1$:

$$\Delta(i + 2^s) = \Big(\mathtt{1} || \langle \mathtt{i} \rangle\Big) \oplus \Big(\mathtt{1} || \langle \mathtt{i+1} \rangle\Big) = \langle \mathtt{i} \rangle \oplus \langle \mathtt{i+1} \rangle = \Delta(i).$$

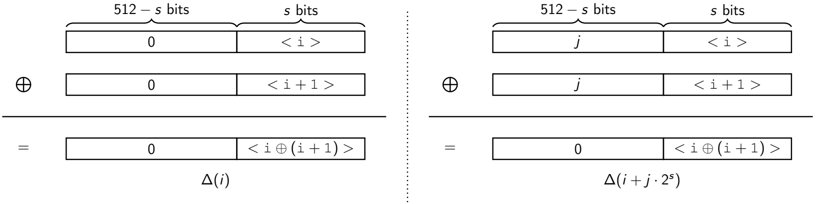
More generally: $F_{\Delta(i)} = F_{\Delta(i + j \cdot 2^s)}$ for all $0 \leq i \leq 2^s - 1$ and $j \geq 0$.

For example, with $s = 2$, F_1 and $F_{1+2^2} = F_5$ are equal.

Relations between functions $F_{\Delta(i)}$ for $1 \leq i \leq t$ (2/2).

Given an integer $s > 0$, we have:

$$\forall i \in \{0, \ldots, 2^s - 2\}, \quad \forall j > 0 : \qquad F_{\Delta(i)} = F_{\Delta(j \cdot 2^s + i)}$$

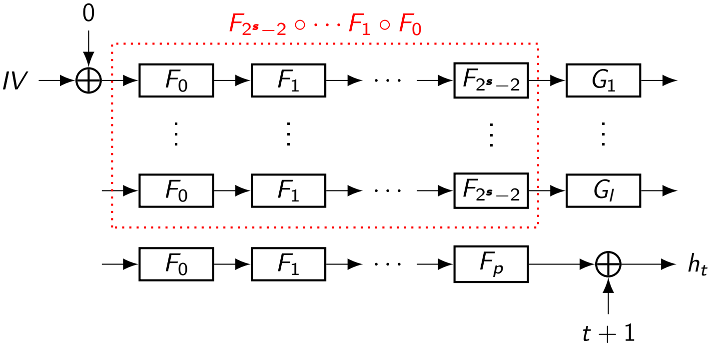


Consequently:

- The same sequence of $2^s - 1$ functions are used in the domain extension algorithm.
- This seems weaker than a true HAIFA mode.

Equivalent description of stage 2 of the domain extension.

- The last function differs in each 2^s-chunk.
  $\Longrightarrow$ We call it $G_j = F_{\Delta(j \times 2^s - 1)}$.
- We define l as the number of $(2^s - 1)$-chains of F functions: $l = \left\lfloor \frac{t}{2^s} \right\rfloor$. Moreover, let p be the remainder of t modulo 2^s.
- That is: the function $F_{2^s-2} \circ \cdots F_1 \circ F_0$ is reused l times.



Cryptographic consequences of the HAIFA instantiation.

`Streebog` is **one** choice of counter usage from the HAIFA framework.

Consequences of this choice:

- Counters at steps i and $i+1$ can be combined.
- Distinction of compression function calls in the HAIFA framework not achieved.
- Domain extension similar to a Merkle-Damgård scheme.
  $\Longrightarrow$ Possibility to apply existing known second-preimage attacks.

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- Domain extension similar to a Merkle-Damgård scheme.
  $\Longrightarrow$ Possibility to apply existing known second-preimage attacks.

Our second-preimage attacks on `Streebog` **(security level: 2^{512}):**

- Using a diamond structure:
  - Original message of at least 2^{179} blocks.
  - 2^{342} compression function evaluations.
- Using a expandable message:
  - Original message of at least 2^{259} blocks.
  - 2^{266} compression function evaluations.

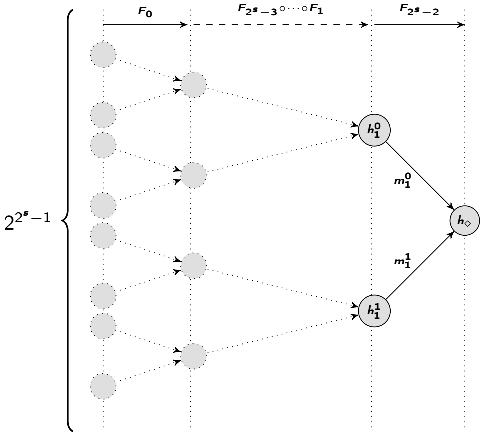
Diamond structure (1/2)

Diamond structure:

- Introduced in [KK06].
- Complete binary tree.
- Nodes: chaining values.
- Edges: 1-block n-bit messages.
- Depth d.

Construction:

- Levels constructed sequentially.
- Complexity: $2^{(n+d)/2}$ calls.
- Evaluation done in [KK13].



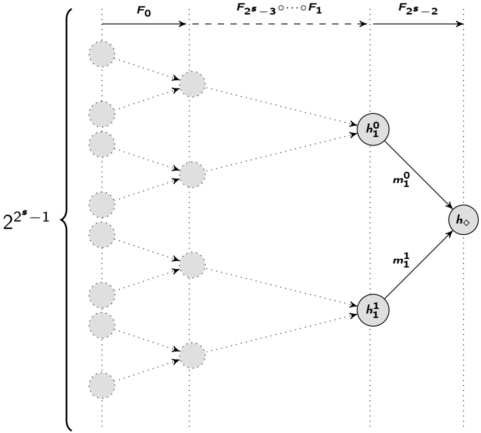
Diamond structure (2/2)

Diamond used in our attack:

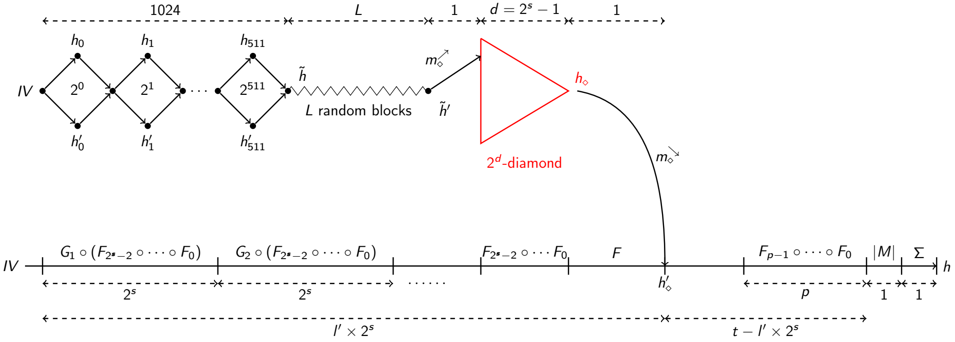
- Root $h_\diamond$.
- Depth $d = 2^s - 1$.
- F_i's used to join the levels.
- #leaves=2^{2^s-1}.

Remarks:

- Same function at each level in the original attack on Merkle-Damgård.
- Here, full control of the counter effect in the $(2^s - 1)$-chains with different functions F_i.



Overview of the diamond attack.



1. Construction of the diamond.
2. Randomize $m_\diamond^\searrow$ to hit $h'_\diamond$.
3. Deduce the counter value N.
4. Construct 2^{512}-multicollision.
5. Randomize L blocks to match $|M|$.
6. Pick about 2^{n-d} $m_\diamond^\nearrow$ to hit the diamond.
7. Evaluate reduced checksum σ.
8. Use multicollision to match $\Sigma - \sigma$.

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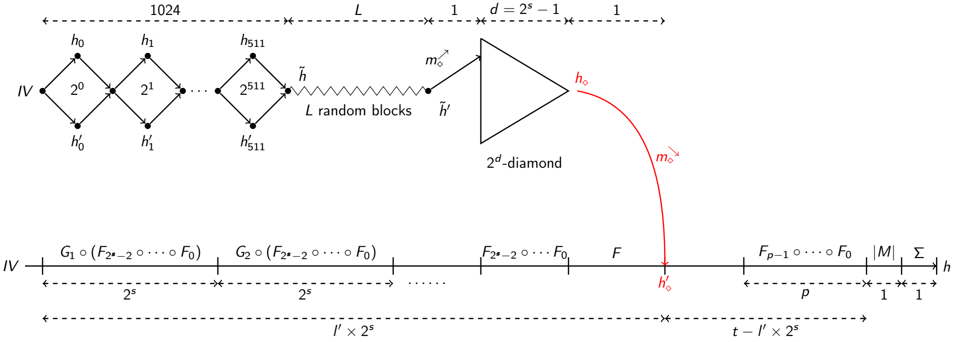

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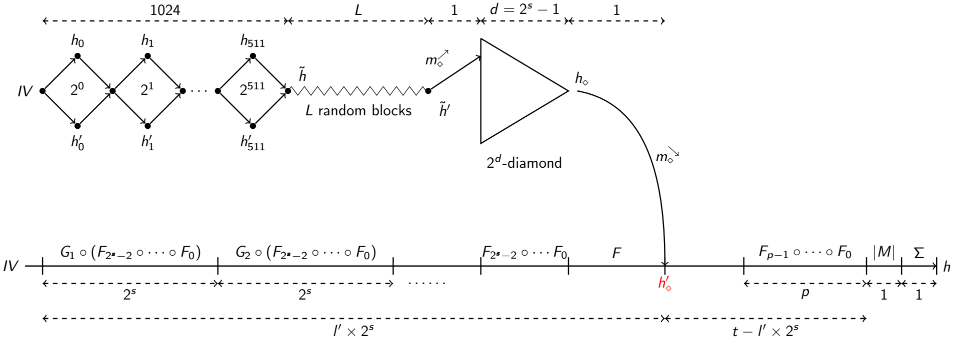

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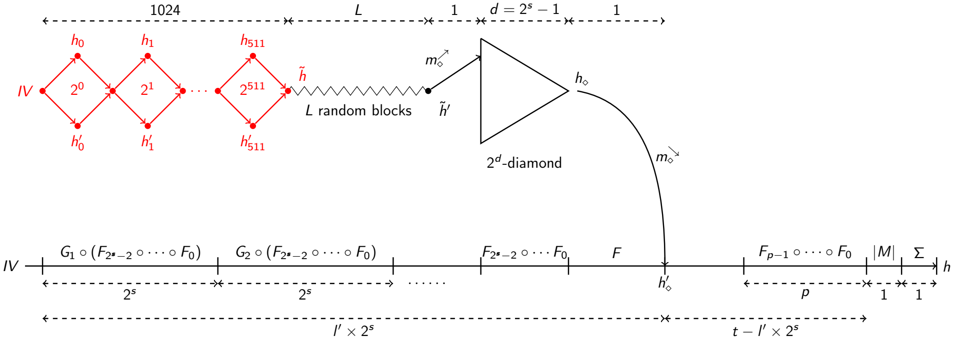

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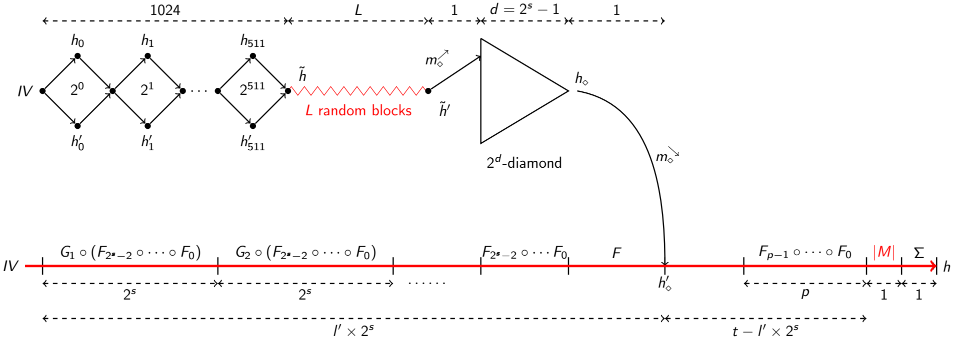

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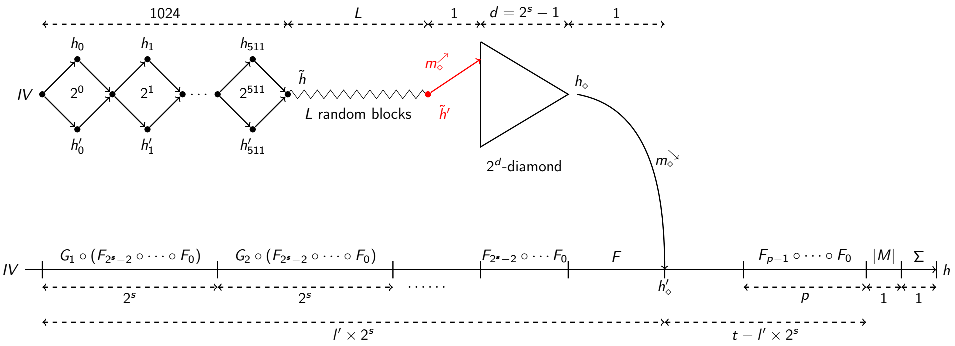

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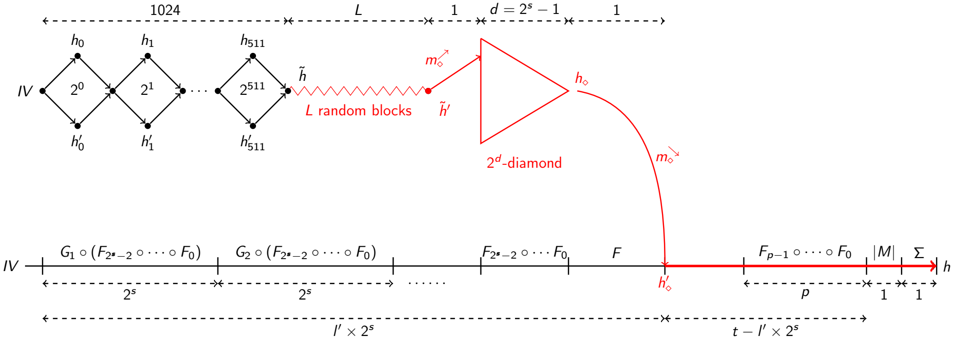

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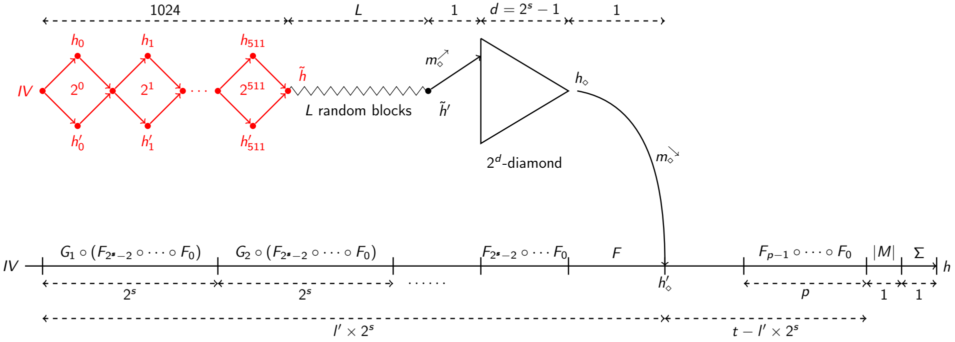

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Complexity analysis of the diamond attack.

Time complexity T

$$T = 2^{(n+d)/2} + 512 \times 2^{n/2} + 2^{n-\log_2(l)} + 2^{n-d},$$

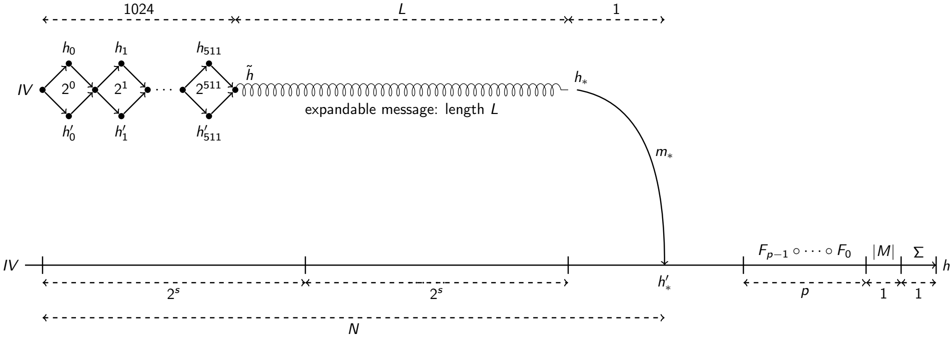
with:

- Construction of the diamond.
- Joux's multicollision using 512 two-block messages.
- Connect the root of the diamond to the original message.
- Connect the multicollision to one leaf of the diamond.

Minimize with:

- $d = n/3 = 2^s - 1$ the depth of the diamond, i.e. $s = \lceil \log_2(n/3) \rceil$.
- as long as $l = \left\lfloor \frac{t}{2^s} \right\rfloor$ is $l \geq 2^{n/3}$, i.e. $t \geq \lceil 2^{n/3+\log_2(n/3)} \rceil$.
- For `Streebog-512`: $T = 2^{342}$ for $|M| \geq 2^{179}$.

Overview of the attack using an expandable message.



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5. Choose the valid length L and solve the checksum.

Complexity analysis.

Time complexity T

$$T = 512 \times 2^{n/2} + 256 \times 2^{n/2} + 2^{n-l},$$

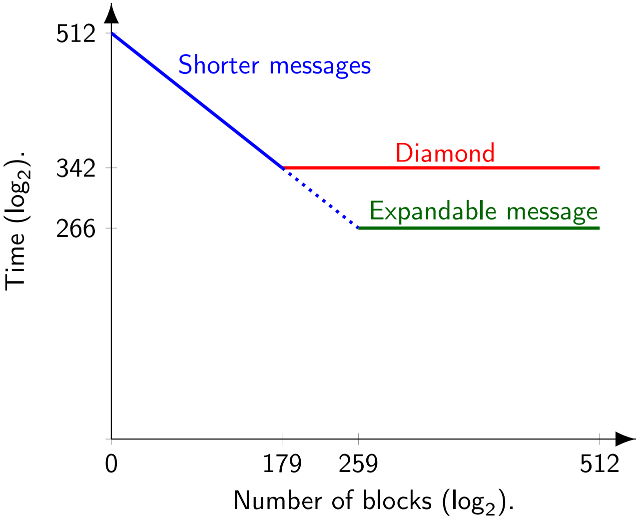
with:

- Joux's multicollision using 512 two-block messages.
- Construction of the expandable message.
- Connect the expandable message to the challenge ($l = \lfloor \frac{t}{2^s} \rfloor$).

Minimize with:

- $l > 2^{n/2}/n$, i.e. more than 2^{259} blocks in the original message.
- T about $n \cdot 2^{n/2}$, i.e. 2^{266} CF evaluations ($s = 11$).

Comparison of the two attacks



Conclusion

- We study Streebog, the Russian hashing standard.
- The hash function instantiates the HAIFA framework.
- We propose an equivalent representation that hijack the counter effect of Streebog-512.
- Consequently, one can reuse previous second-preimage attack strategies:
  - using a diamond structure,
  - using an expandable message.
- The two attacks have time complexity T for message length $> L$:
  - $T = 2^{342}$ and $L = 2^{179}$,
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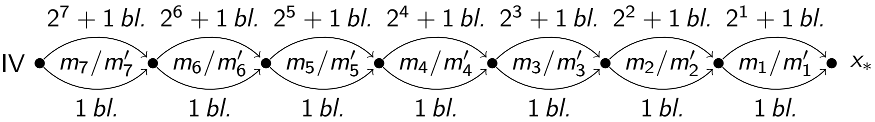
Thank you!

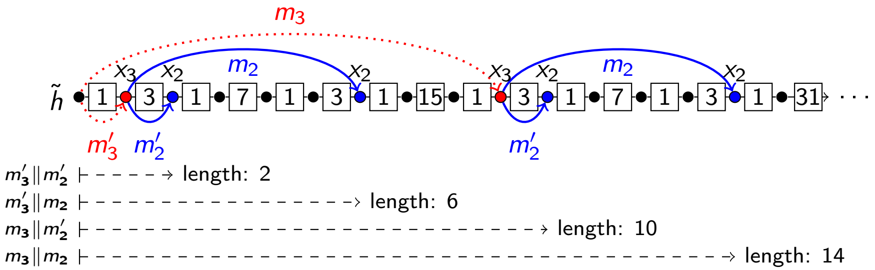
Expandable message

- Expandable messages due to [KS05]
- Multicollision with different lengths:
  - t pairs with lengths $(1, 2^k + 1)$, $0 \leq k < t$.
  - Set of 2^t messages with length in $[t, 2^t + t - 1]$.
  - All reach the same final chaining value x_*.
- Construction of a message m of length $t + L$ using the binary representation of L, that link IV to x_*.
- Second-preimage attack on MD:
  - Link x_* to original message using random blocks.
  - This gives the length to use in the expandable message.
  - HAIFA prevents using an expandable message with the counter input.

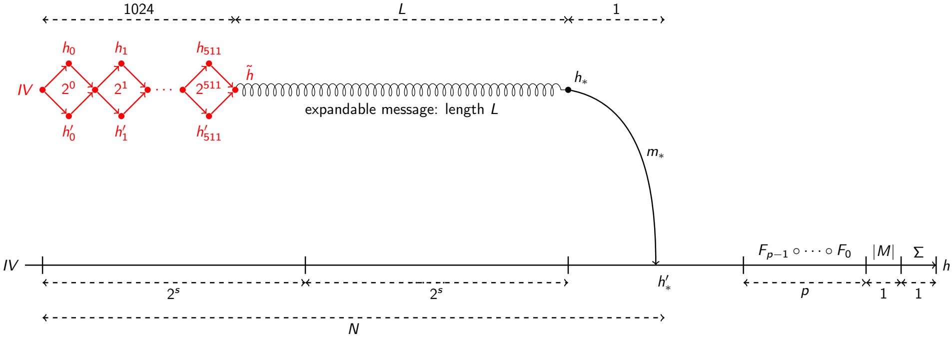


Expandable messages in Streebog

- Here, the counter input is weak.
- We can still apply the expandable message technique:
  - The functions $F_{\Delta(i)}$ are independent of the counter,
  - but the inner calls are not the same (HAIFA, not MD).
- Small example: 4 messages from $\tilde{h}$ to x_2.
  - Find (m'_3, m_3) of lengths $(1, 2^3+1)$ colliding on x_3.
  - Find (m'_2, m_2) of lengths $(1, 2^2+1)$ colliding on x_2.
  - The 4-message structure has lengths in $\{2, 6, 10, 14\}$.



Overview of the attack using an expandable message.



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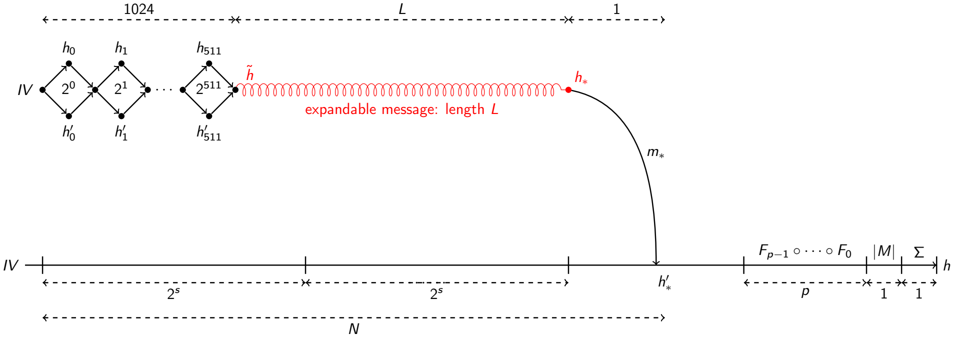

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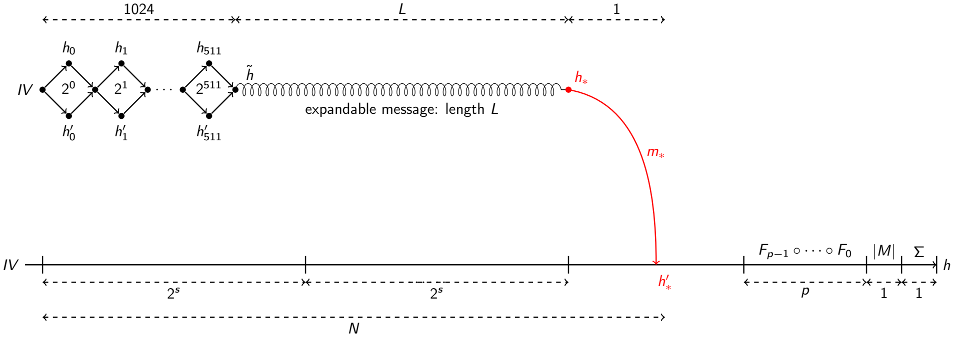

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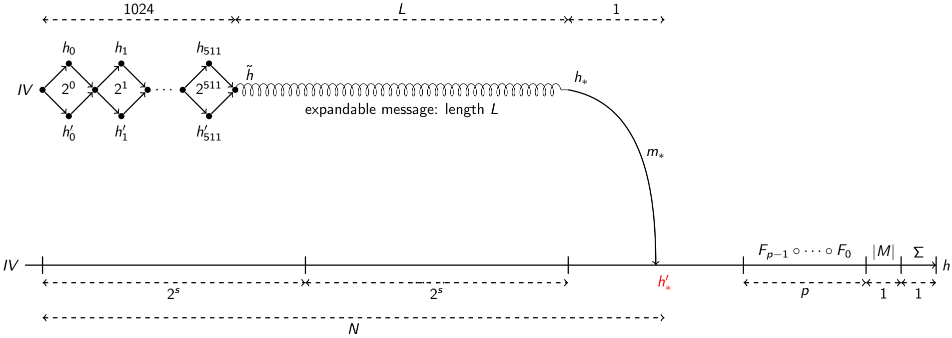

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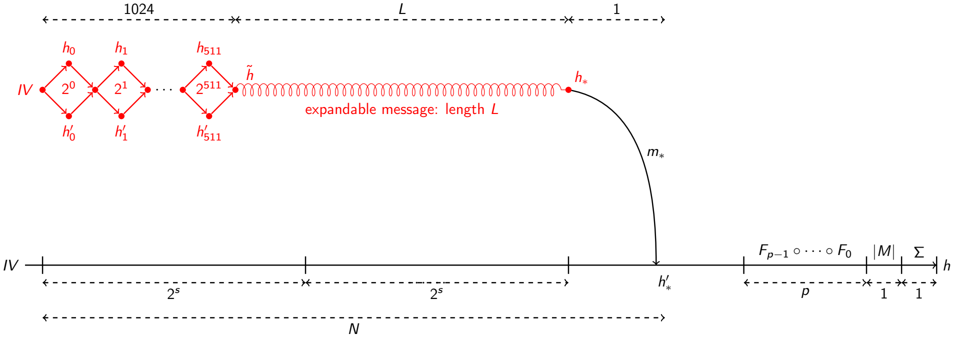

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3. Randomize m_* to hit h'_*.
4. Deduce the counter value.
5. Choose the valid length L and solve the checksum.

Complexity analysis.

Time complexity T

$$T = 512 \times 2^{n/2} + 256 \times 2^{n/2} + 2^{n-l},$$

with:

- Joux's multicollision using 512 two-block messages.
- Construction of the expandable message.
- Connect the expandable message to the challenge ($l = \lfloor \frac{t}{2^s} \rfloor$).

Minimize with:

- $l > 2^{n/2}/n$, i.e. more than 2^{259} blocks in the original message.
- T about $n \cdot 2^{n/2}$, i.e. 2^{266} CF evaluations ($s = 11$).

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