Hash Functions and the (Amplified) Boomerang Attack

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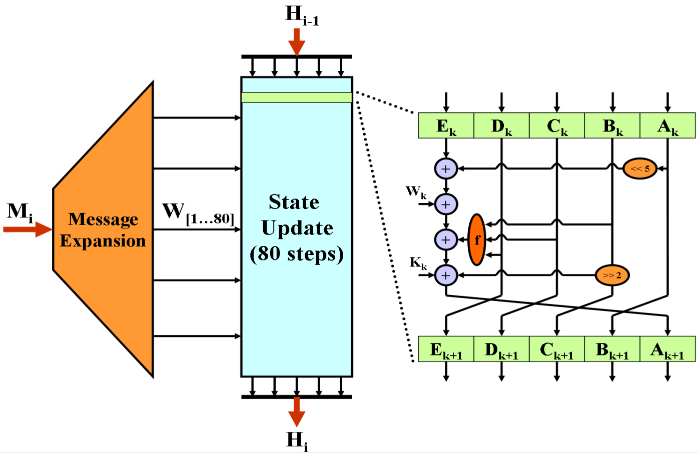
Outline

Outline

The SHA-1 hash function (1)

Merkle-Damgård + Davies-Meyer mode.



The SHA-1 hash function (2)

Message expansion:

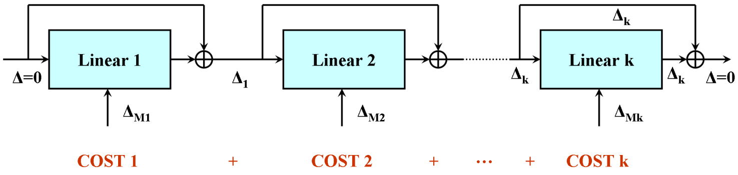
$$W_i = \begin{cases} M_i, & \text{for } 0 \leq i \leq 15 \\ (W_{i-3} \oplus W_{i-8} \oplus W_{i-14} \oplus W_{i-16}) \lll 1, & \text{for } 16 \leq i \leq 79 \end{cases}$$

Boolean functions:

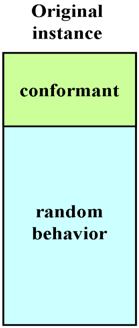
step i	$f_i(B, C, D)$
$1 \leq i \leq 20$	$f_{IF} = (B \wedge C) \oplus (\overline{B} \wedge D)$
$21 \leq i \leq 40$	$f_{XOR} = B \oplus C \oplus D$
$41 \leq i \leq 60$	$f_{MAJ} = (B \wedge C) \oplus (B \wedge D) \oplus (C \wedge D)$
$61 \leq i \leq 80$	$f_{XOR} = B \oplus C \oplus D$

Collision attack against SHA-0 (Biham et al.)

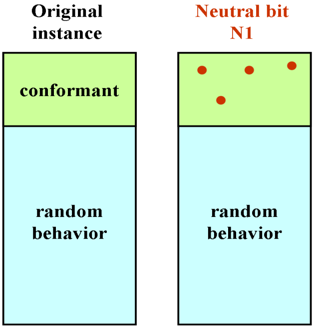
- local collision: insert a perturbation and correct it! Then find perturbation and corrections vectors such that the overall difference mask satisfies the message expansion.
- multi-block technique: you can use several blocks to find a collision.



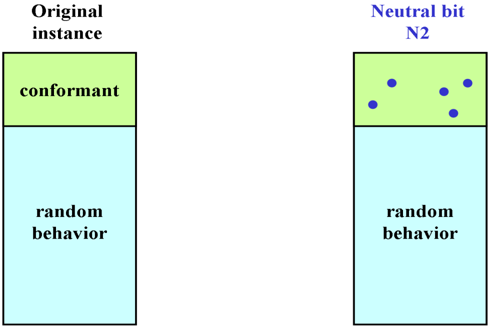
The neutral bits



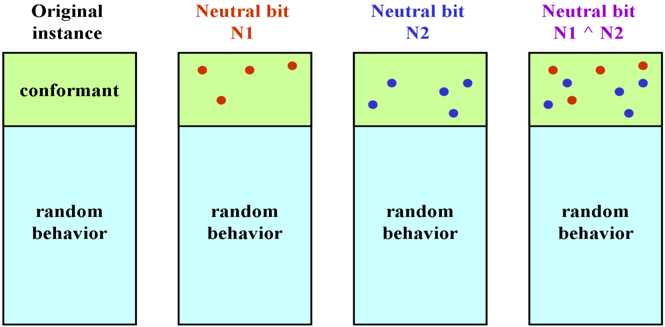
The neutral bits



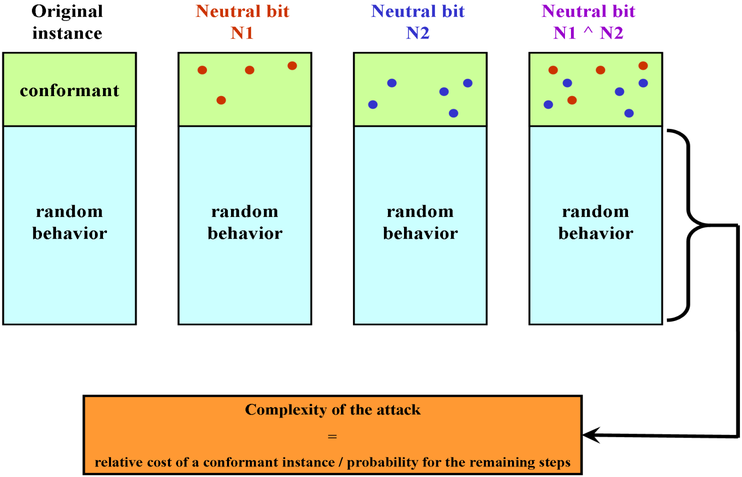
The neutral bits



The neutral bits

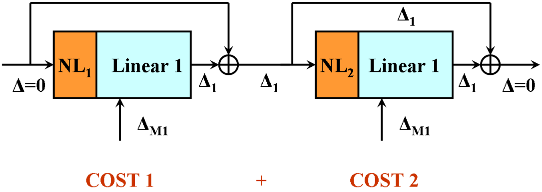


The neutral bits



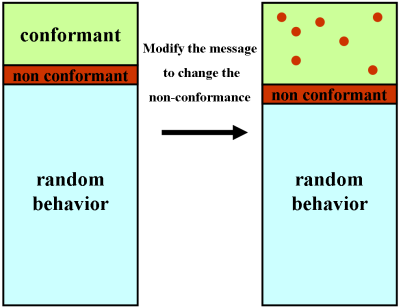
Collision attack against SHA-1 (Wang et al.)

- modify (by hand!) the first steps of the differential path $\Longrightarrow$ non-linear part.
- find (by hand!) the sufficient conditions such that everything goes as expected
  $\Longrightarrow$ evaluate the probability of the differential path.
- 2^{69} message modifications (improved to 2^{63} but not published) [*Wang*, *Yin*, *Yu* – 2005].

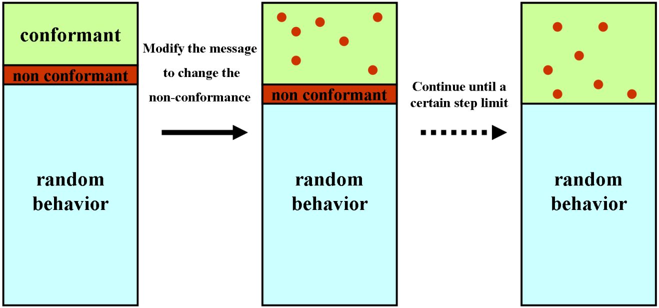


Wang et al.'s attacks: the message modifications

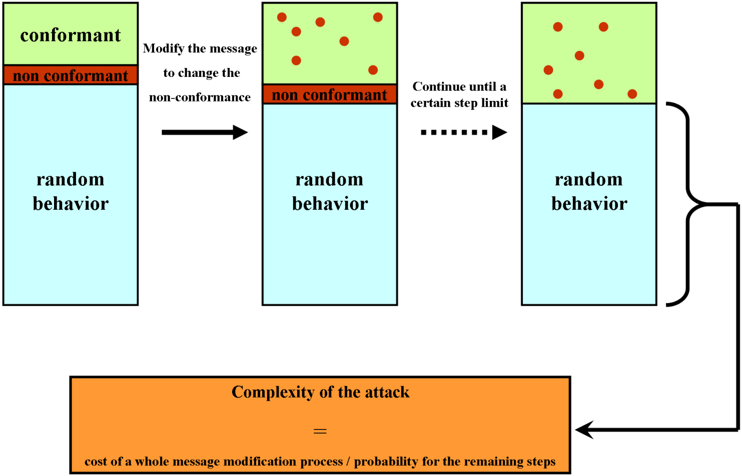
Wang et al.'s attacks: the message modifications



Wang et al.'s attacks: the message modifications



Wang et al.'s attacks: the message modifications



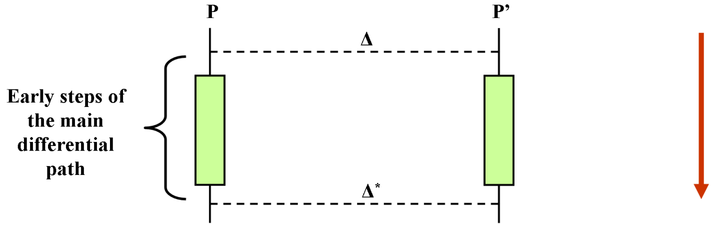
New attacks

Wang et al. found everything by hand! Can we provide more theoretical explanations of what is happening ?

- a better way of evaluating the probability of a diff. path [*De Cannière*, *Rechberger* – 2006].
- automatic and heuristic search of non linear parts [*De Cannière*, *Rechberger* – 2006].
- finding sufficient conditions with Gröbner Basis [*Sugita*, *Kawazoe*, *Imai* – 2007].
- finding message modifications with Gröbner Basis [*Sugita*, *Kawazoe*, *Imai* – 2007].
- a 70-step collision [*De Cannière*, *Mendel*, *Rechberger* – 2007].

Outline

The (amplified) boomerang attack for hash functions (1)



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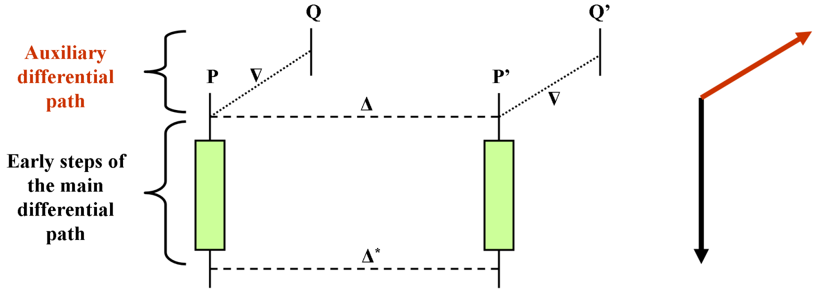
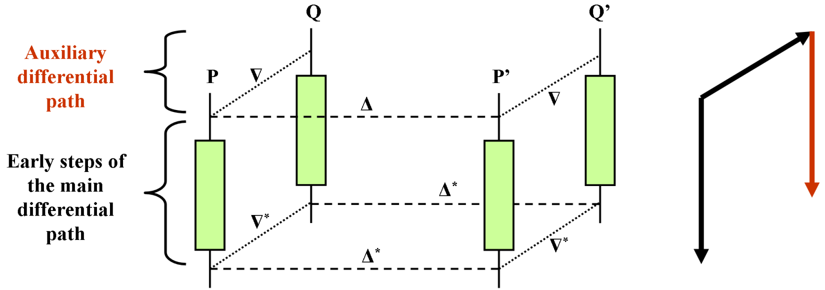
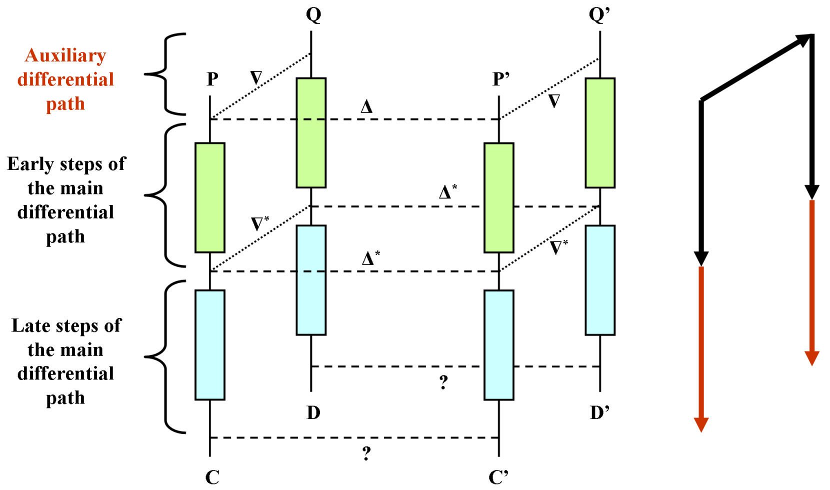


The (amplified) boomerang attack for hash functions (1)



The (amplified) boomerang attack for hash functions (1)



The (amplified) boomerang attack for hash functions (2)

Two possibilities of use:

- neutral bits/message modification approach: instantiate a message pair and check if there is good auxiliary differential paths
  $\Longrightarrow$ generalization of neutral bits/message modification.
- explicit conditions approach: BEFORE instantiating the message pair, fix some bits so that you will be sure that very good auxiliary differential paths exist
  $\Longrightarrow$ allows you to find very powerful neutral bits/message modification!

In neutral bits setting: for t auxiliary differential paths, you get 2^t conformant pairs of messages for free (with an independence assumption, true in practice).

Outline

A useful tool: the local collision

$$A_{i+1} = (A_i \lll 5) + f_i(A_{i-1}, A_{i-2} \ggg 2, A_{i-3} \ggg 2) + (A_{i-4} \ggg 2) + K_i + W_i.$$

step	type	constraints
$i+1$	no carry	$W_i^j = a$, $A_{i+1}^j = a$

i	A_i	W_i
-1:	--------------------------------	
00:	--------------------------------	-----------------------------a--
01:	-----------------------------a--	--------------------------------
02:	--------------------------------	--------------------------------
03:	--------------------------------	--------------------------------
04:	--------------------------------	--------------------------------
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

$$A_{i+1} = (A_i \ll 5) + f_i(A_{i-1}, A_{i-2} \gg 2, A_{i-3} \gg 2) + (A_{i-4} \gg 2) + K_i + W_i.$$

step	type	constraints
$i+1$	no carry	$W_i^j = a$, $A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$

i	A_i	W_i
-1:	--------------------------------	
00:	--------------------------------	-----------------------------a--
01:	-----------------------------a--	------------------------$\bar{a}$-------
02:	--------------------------------	--------------------------------
03:	--------------------------------	--------------------------------
04:	--------------------------------	--------------------------------
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

$$A_{i+1} = (A_i \lll 5) + f_i(A_{i-1}, A_{i-2} \ggg 2, A_{i-3} \ggg 2) + (A_{i-4} \ggg 2) + K_i + W_i.$$

step	type	constraints
$i+1$	no carry	$W_i^j = a$, $A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
	correction	$A_{i-1}^{j+2} \neq A_i^{j+2}$, $W_{i+2}^j = \bar{a}$

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	-----------------------------a--	------------------------ā-------
02:	--------------------------------	--------------------------------
03:	--------------------------------	--------------------------------
04:	--------------------------------	--------------------------------
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

$$A_{i+1} = (A_i \lll 5) + f_i(A_{i-1}, A_{i-2} \ggg 2, A_{i-3} \ggg 2) + (A_{i-4} \ggg 2) + K_i + W_i.$$

step	type	constraints
$i+1$	no carry	$W_i^j = a,\ A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
$i+4$	no correction	$A_{i+2}^{j-2} = 0$
	correction	$A_{i+2}^{j-2} = 1,\ W_{i+3}^{j-2} = \bar{a}$

i	A_i	W_i
-1:	`---------------------------d----`	
00:	`---------------------------d----`	`-----------------------------a--`
01:	`-----------------------------a--`	`------------------------ā-------`
02:	`-------------------------------1`	`--------------------------------`
03:	`--------------------------------`	`-------------------------------ā`
04:	`--------------------------------`	`--------------------------------`
05:	`--------------------------------`	`--------------------------------`
06:	`--------------------------------`	`--------------------------------`

A useful tool: the local collision

$$A_{i+1} = (A_i \lll 5) + f_i(A_{i-1}, A_{i-2} \ggg 2, A_{i-3} \ggg 2) + (A_{i-4} \ggg 2) + K_i + W_i .$$

step	type	constraints
$i+1$	no carry	$W_i^j = a,\ A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
$i+4$	correction	$A_{i+2}^{j-2} = 1,\ W_{i+3}^{j-2} = \bar{a}$
$i+5$	no correction	$A_{i+3}^{j-2} = 1$
	correction	$A_{i+3}^{j-2} = 0,\ W_{i+4}^{j-2} = \bar{a}$

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	-----------------------------a--	------------------------$\bar{a}$-------
02:	-------------------------------1	--------------------------------
03:	-------------------------------0	-------------------------------$\bar{a}$
04:	--------------------------------	-------------------------------$\bar{a}$
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

$$A_{i+1} = (A_i \lll 5) + f_i(A_{i-1}, A_{i-2} \ggg 2, A_{i-3} \ggg 2) + (A_{i-4} \ggg 2) + K_i + W_i.$$

step	type	constraints
$i+1$	no carry	$W_i^j = a,\ A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
$i+4$	correction	$A_{i+2}^{j-2} = 1,\ W_{i+3}^{j-2} = \bar{a}$
$i+5$	correction	$A_{i+3}^{j-2} = 0,\ W_{i+4}^{j-2} = \bar{a}$
$i+6$	correction	$W_{i+5}^{j-2} = \bar{a}$

i	A_i	W_i
-1:	`---------------------------d----`	
00:	`---------------------------d----`	`-----------------------------a--`
01:	`-----------------------------a--`	`------------------------ā-------`
02:	`-------------------------------1`	`--------------------------------`
03:	`-------------------------------0`	`-------------------------------ā`
04:	`--------------------------------`	`-------------------------------ā`
05:	`--------------------------------`	`-------------------------------ā`
06:	`--------------------------------`	`--------------------------------`

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
perturbation mask	1010000000100000	
differences on W^j	1010000000100000	0000000010110110
differences on W^{j+5}	0101000000010000	0000000001011011
differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------$\bar{a}$-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------$\bar{b}$------$\bar{a}$
04:	-------------------------------0	-------------------------------$\bar{a}$
05:	-------------------------------0	-------------------------------$\bar{a}$
06:	--------------------------------	-------------------------------$\bar{b}$
07:	--------------------------------	-------------------------------$\bar{b}$
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------$\bar{c}$-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------$\bar{c}$
15:	--------------------------------	-------------------------------$\bar{c}$

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
perturbation mask	1010000000100000	
differences on W^j	1010000000100000	0000000010110110
differences on W^{j+5}	0101000000010000	0000000001011011
differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------ā-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------b̄------ā
04:	-------------------------------0	-------------------------------ā
05:	-------------------------------0	-------------------------------ā
06:	--------------------------------	-------------------------------b̄
07:	--------------------------------	-------------------------------b̄
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------c̄-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------c̄
15:	--------------------------------	-------------------------------c̄

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
perturbation mask	1010000000100000	
differences on W^j	1010000000100000	0000000010110110
differences on W^{j+5}	0101000000010000	0000000001011011
differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------$\bar{a}$-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------$\bar{b}$------$\bar{a}$
04:	-------------------------------0	-------------------------------$\bar{a}$
05:	-------------------------------0	-------------------------------$\bar{a}$
06:	--------------------------------	-------------------------------$\bar{b}$
07:	--------------------------------	-------------------------------$\bar{b}$
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------$\bar{c}$-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------$\bar{c}$
15:	--------------------------------	-------------------------------$\bar{c}$

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
perturbation mask	1010000000100000	
differences on W^j	1010000000100000	0000000010110110
differences on W^{j+5}	0101000000010000	0000000001011011
differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------$\bar{a}$-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------$\bar{b}$------$\bar{a}$
04:	-------------------------------0	-------------------------------$\bar{a}$
05:	-------------------------------0	-------------------------------$\bar{a}$
06:	--------------------------------	-------------------------------$\bar{b}$
07:	--------------------------------	-------------------------------$\bar{b}$
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------$\bar{c}$-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------$\bar{c}$
15:	--------------------------------	-------------------------------$\bar{c}$

Placing auxiliary differential paths

i	A_i	W_i
-4:	00101001010011011100100101000111	
-3:	00000111100001000110010101100010	
-2:	11011000010000101001111101011111	
-1:	01011011110111101101101111010001	
00:	01000010101101110111101110011011	1uu11101100111110110--0111111011
01:	n1n010111001011001001-0100100110	nuu101-10001011--111111101u1n0n1
02:	1nu11--01111101111101101111111u1	--n11-----0-10-1111000110n0111uu
03:	nnu00-----0-00-0110000110111110n	x-nn-1--1--01010001001--1u111001
04:	u010u11-0--00010010110-1010un0u1	uu-u0-------11-0--1011001n1n10nu
05:	1001u00-0--000000000001u00011010	nn-u0------11010111--1--11n100u1
06:	011unnnnnnnnnnnnnnn1---110n001uu	00n-------1-1--1--00111100011001
07:	u110-01000000u010110nu111uu1010n	1nu001------1--1-100-1-10-un-0n-
08:	1111010111111---011unu110-0--nu1	-un0----------11---------u0111nu
09:	-0010---1--1--01-0u-10nnnnu01010	--u0-------------1--1001-u1--100
10:	--------1--1--0--01-101nu1111u10	xxu00-----0--1--1--0--1--u----n-
11:	0---------0--1--1--0n-100nn0u1n0	-xn--1--0--0--1--0---11-0010--x-
12:	0---0-------0--0--0--01-010n1-nn	x------------------------------u
13:	00----------0--0--0--00100n0n-00	--10------------------0--1n1----
14:	-0--0-----------------10001u0un-	---1--------1--0--0--1--000---xn
15:	n-----------------------unnn1101	-x-10-------1--0--0--1--0u-n--u-
16:	--1---------------------1--nu001	-n0---------------------1u0-----
17:	n-0-----------------------111-0n	xxn-----------------1---1u-x--n-
18:	-11-------------------------101-	x-u1--------------------0----0--
19:	------------------------------u-	x----------------------11n------
20:	--------------------------------	--x----------------------------x
	...	...

Theoretical Result

- we can use boomerang attacks in addition with neutral bits or known message modifications if we carefully check that the auxiliary paths remain valid.
- message modifications can be costly and the 2^{63} attack is not yet published.
- works well with neutral bits.
- we expect an improvement of a factor 32 (5 auxiliary paths) on the known attacks against SHA-1 with 80 steps.

If you are interested in the details, see our paper!

Practical Result: 70-step collision

A 2-block collision attack against 70-step SHA-1 in number of compression function calls to an efficient implementation of SHA-1 (openSSL).

	De Cannière et al. (2007)	Boomerang attack with 5 auxiliary paths
1st block	2^{41}	$2^{36,5}$
2nd block	2^{44}	2^{39}

A 70-step collision for SHA-1 took us less than 10 hours of computation on a cluster of 8 computers !

The 70-step collision

i	Message 1 - First Block	Message 2 - First Block
0-3	BDD77848 4FF53120 678B09E0 6C08A508	2DD77838 FFF53173 578B09E8 6C08A54B
4-7	950A1CB9 3A92154B B78CA6D8 1092006C	450A1CCB 8A92155B 478CA6BA D092002E
8-11	A3C3331B 9CE9568E 1D629EB0 7051A403	A3C3332B 7CE956CC 3D629ED0 9051A442
12-15	F04FC758 3BBE0731 76C54123 8A00A65A	D04FC708 FBBE0770 96C54151 2A00A659

i	Message 1 - Second Block	Message 2 - Second Block
0-3	A77D4037 5E854D1E 0425118C 8D5788C3	377D4047 EE854D4D 34251184 8D578880
4-7	3117F80B 300B5150 4EF7758D A4F02975	E117F879 800B5140 BEF775EF 64F02937
8-11	B4237099 9A7E7BB8 3EFFF106 DFFE9648	B42370A9 7A7E7BFA 1EFFF166 3FFE9609
12-15	D8EC1118 4A3C66FC A9FD35D5 4E6E26CC	F8EC1148 8A3C66BD 49FD35A7 EE6E26CF

Final Hash Value
8F2FB5E0 EA262496 653A9B0E 23D75B12 B936129B

Outline

Yet another way of using freedom degrees ...

- boomerang attack for hash functions is nothing more than another way of cleverly using the freedom degrees from the message.
- message modifications, neutral bits, Klima's tunnels for MD5, auxiliary differentials are closely related.
- generally speaking they all have pros and cons:

	message modifications	neutral bits	small auxiliary paths	big auxiliary paths
speed cost	big	medium	small	small
freedom degrees cost	medium	small	small	big
range	medium	small	small	long

... but freedom degrees are not unlimited!

- twofold waste of freedom degrees: or we use a lot of freedom degrees for a small gain, or some freedom degrees are left unused.
- it would be great to find a way to use exactly what we need from all those techniques.
- not trivial since we need to settle the long range characteristics first, which imposes a lot (too much ?) of constraints.
- maybe a further generalization of those techniques may achieve this ?

Thank you!