Hash Functions and the Boomerang Attack

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Outline

1. Introduction
2. The (Amplified) Boomerang Attack
3. Application to SHA-1
4. Conclusion

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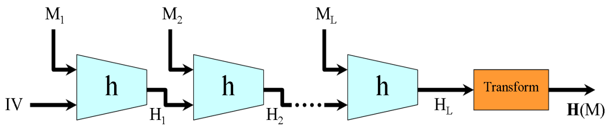
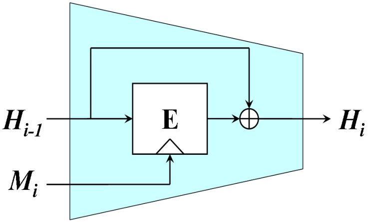
The MDx-SHAx family of hash functions: high level design

message + padding = M = $M_1 \| M_2 \| \ldots \| M_L$



Merkle-Damgård

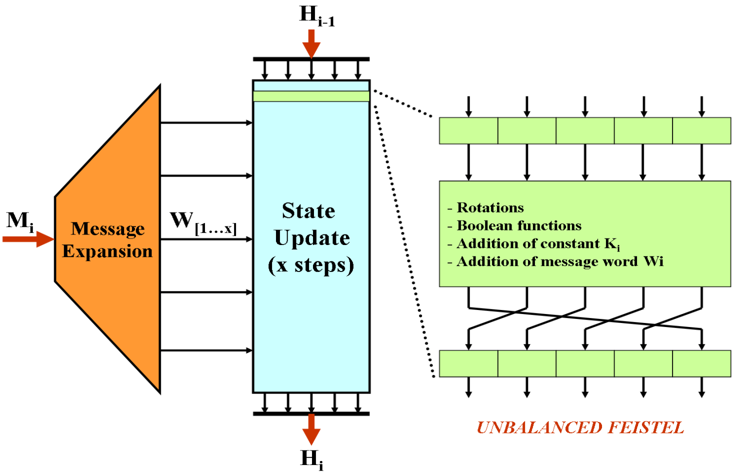
+



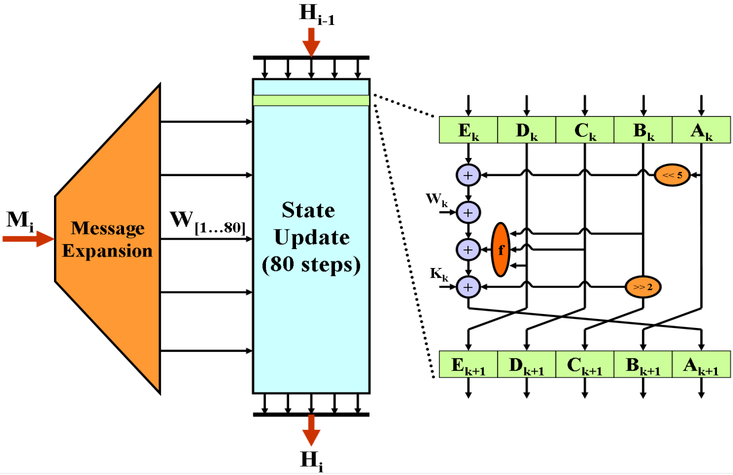
Davies-Meyer

Mode

The MDx-SHAx family of hash functions: the internal block cipher



The SHA-1 compression function (1)



The SHA-1 compression function (2)

Message expansion:

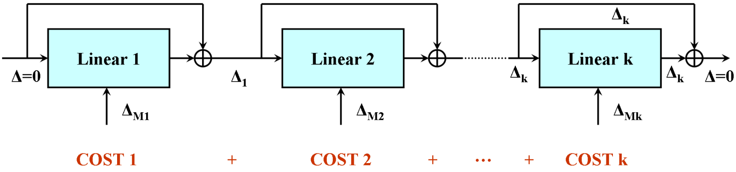
$$W_i = \begin{cases} M_i, & \text{for } 0 \leq i \leq 15 \\ (W_{i-3} \oplus W_{i-8} \oplus W_{i-14} \oplus W_{i-16}) \lll 1, & \text{for } 16 \leq i \leq 79 \end{cases}$$

Boolean functions:

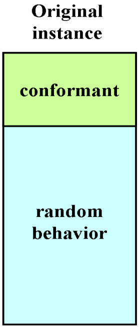
round	step i	$f_i(B, C, D)$
1	$1 \leq i \leq 20$	$f_{IF} = (B \wedge C) \oplus (\overline{B} \wedge D)$
2	$21 \leq i \leq 40$	$f_{XOR} = B \oplus C \oplus D$
3	$41 \leq i \leq 60$	$f_{MAJ} = (B \wedge C) \oplus (B \wedge D) \oplus (C \wedge D)$
4	$61 \leq i \leq 80$	$f_{XOR} = B \oplus C \oplus D$

Chabaud-Joux method for collision attack against SHA-0

- local collision: insert a perturbation and correct it !
- find perturbation and corrections vectors such that the overall difference mask verifies the message expansion.
- you can use several blocks to find a collision:



The neutral bits



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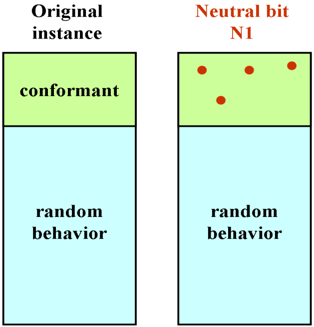

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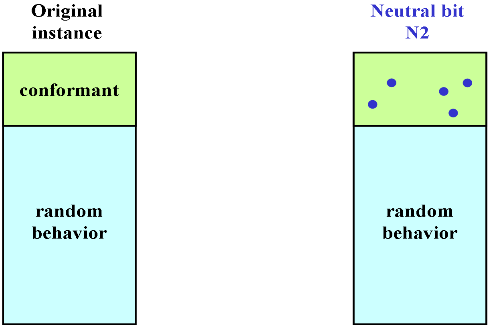

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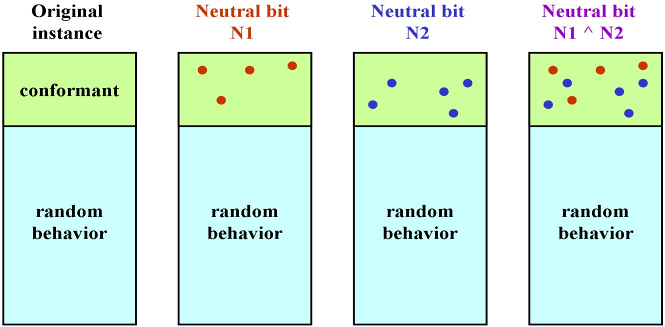

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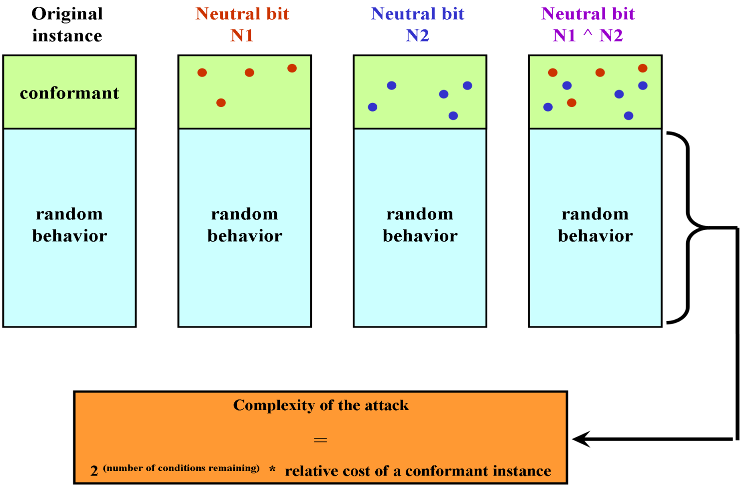
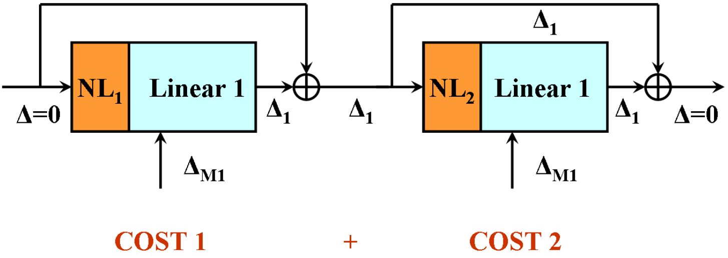


Wang et al.'s attacks: the differential path

- modify (by hand !) the first steps of the differential path $\Longrightarrow$ non-linear part.
- find (by hand !) the necessary conditions such that everything goes as expected $\Longrightarrow$ gives a lower bound on the probability of the differential path.



Wang et al.'s attacks: the message modifications

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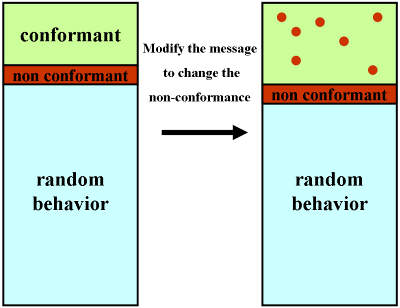
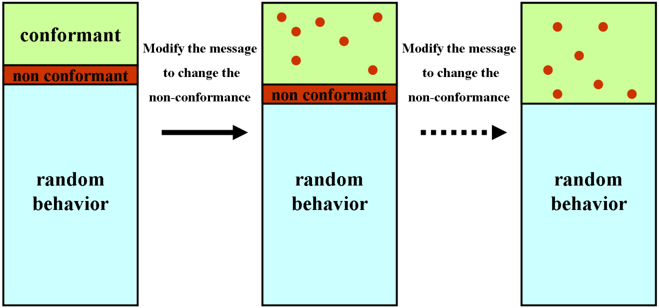


Wang et al.'s attacks: the message modifications



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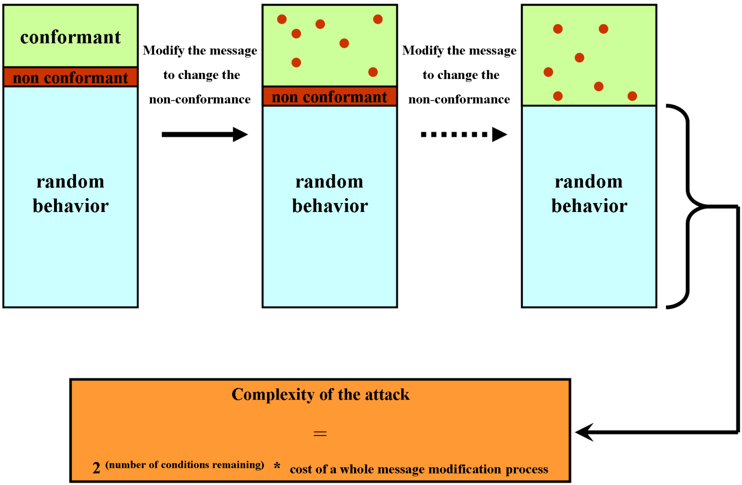

New attacks

Wang et al. found everything by hand ! Can we provide most ”theoretical” explanations of what is happening ?

- a better way of evaluating the probability of a diff. path [*DeCannière*, *Rechberger* – 2006].
- automatic and heuristic search of non linear parts [*De Cannière*, *Rechberger* – 2006].
- finding sufficient conditions with Gröbner Basis [*Sugita*, *Kawazoe*, *Imai* – 2007].
- finding message modifications with Gröbner Basis [*Sugita*, *Kawazoe*, *Imai* – 2007].

Results of known attacks

- 2^{69} message modifications (improved to 2^{63} but not published)
  [*Wang*, *Yin*, *Yu* – 2005].
- ... but message modifications can cost a lot !
  [*Sugita*, *Kawazoe*, *Imai* – 2007].
- fast collisions for 58 steps
  [*Sugita*, *Kawazoe*, *Imai* – 2007].
- a 70-step collision
  [*DeCannière*, *Rechberger* – 2006].

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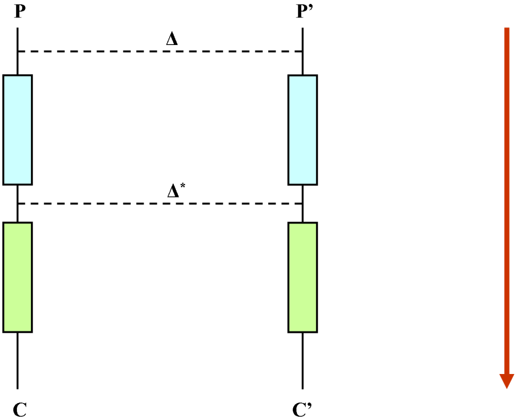
The boomerang attack: [*Wagner* – 1999]



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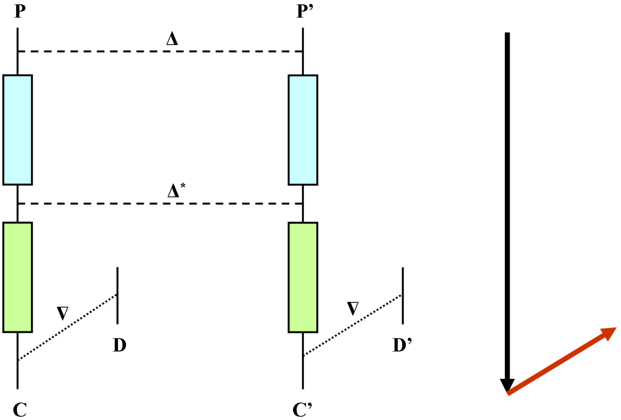

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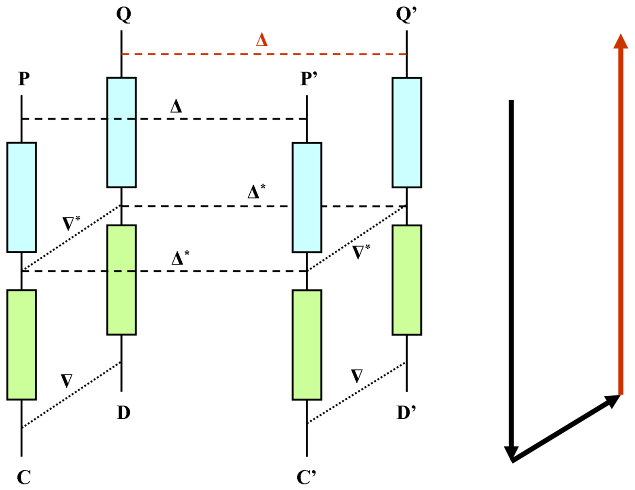
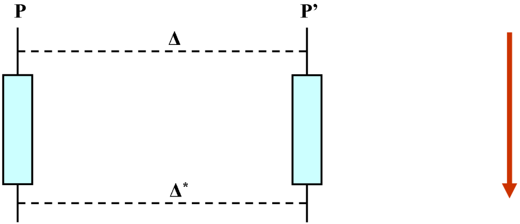


The (amplified) boomerang attack for hash functions (1)



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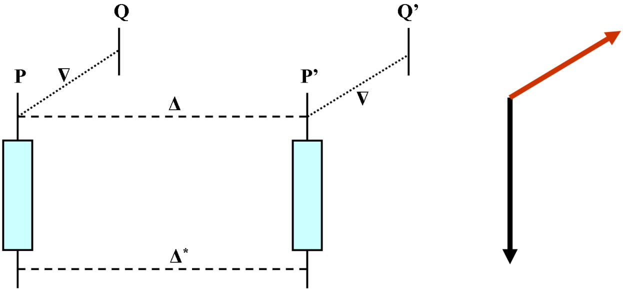

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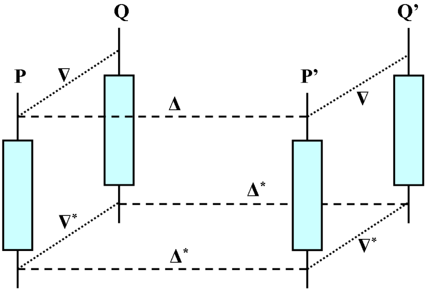

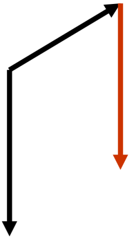

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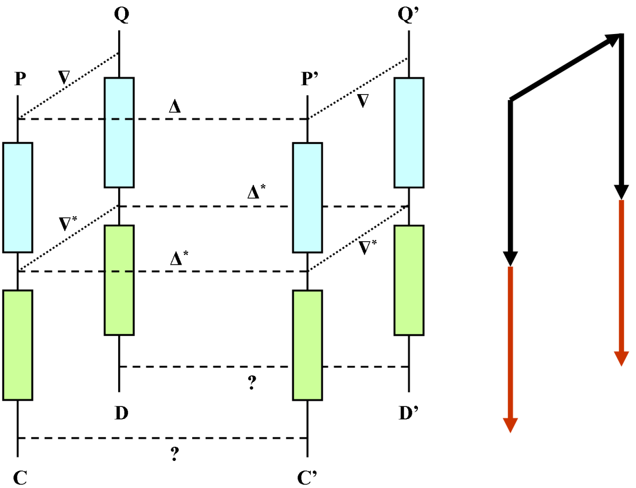

The (amplified) boomerang attack for hash functions (2)

We call the small differential path auxiliary differential path.

Two possibilities of use:

- neutral bits approach: instantiate a message pair and check is there is good auxiliary differential paths $\Longrightarrow$ generalization of neutral bits.
- explicit conditions approach: before instantiating the message pair, fix some bits so that you will be sure that very good auxiliary differential paths exist $\Longrightarrow$ allows you to find very powerful neutral bits !

For t auxiliary differential paths, you get 2^t conformant pairs of messages for free (with an independence assumption, true in practice).

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A useful tool: the local collision

step	type	constraints
$i+1$	no carry	$W_i^j = a$, $A_{i+1}^j = a$

i	A_i	W_i
-1:	--------------------------------	
00:	--------------------------------	-----------------------------a--
01:	-----------------------------a--	--------------------------------
02:	--------------------------------	--------------------------------
03:	--------------------------------	--------------------------------
04:	--------------------------------	--------------------------------
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

step	type	constraints
$i+1$	no carry	$W_i^j = a$, $A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$

i	A_i	W_i
-1:	--------------------------------	
00:	--------------------------------	-----------------------------a--
01:	-----------------------------a--	------------------------$\bar{a}$-------
02:	--------------------------------	--------------------------------
03:	--------------------------------	--------------------------------
04:	--------------------------------	--------------------------------
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

step	type	constraints
$i+1$	no carry	$W_i^j = a$, $A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
	correction	$A_{i-1}^{j+2} \neq A_i^{j+2}$, $W_{i+2}^j = \bar{a}$

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	-----------------------------a--	------------------------ā-------
02:	--------------------------------	--------------------------------
03:	--------------------------------	--------------------------------
04:	--------------------------------	--------------------------------
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

step	type	constraints
$i+1$	no carry	$W_i^j = a$, $A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
$i+4$	no correction	$A_{i+2}^{j-2} = 0$
	correction	$A_{i+2}^{j-2} = 1$, $W_{i+3}^{j-2} = \bar{a}$

i	A_i	W_i
-1:	`---------------------------d----`	
00:	`---------------------------d----`	`-----------------------------a--`
01:	`-----------------------------a--`	`------------------------ā-------`
02:	`-------------------------------1`	`--------------------------------`
03:	`--------------------------------`	`-------------------------------ā`
04:	`--------------------------------`	`--------------------------------`
05:	`--------------------------------`	`--------------------------------`
06:	`--------------------------------`	`--------------------------------`

A useful tool: the local collision

step	type	constraints
$i+1$	no carry	$W_i^j = a,\ A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
$i+4$	correction	$A_{i+2}^{j-2} = 1,\ W_{i+3}^{j-2} = \bar{a}$
$i+5$	no correction	$A_{i+3}^{j-2} = 1$
	correction	$A_{i+3}^{j-2} = 0,\ W_{i+4}^{j-2} = \bar{a}$

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	-----------------------------a--	------------------------$\bar{a}$-------
02:	-------------------------------1	--------------------------------
03:	-------------------------------0	-------------------------------$\bar{a}$
04:	--------------------------------	-------------------------------$\bar{a}$
05:	--------------------------------	--------------------------------
06:	--------------------------------	--------------------------------

A useful tool: the local collision

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$i+1$	no carry	$W_i^j = a,\ A_{i+1}^j = a$
$i+2$	correction	$W_{i+1}^{j+5} = \bar{a}$
$i+3$	no correction	$A_{i-1}^{j+2} = A_i^{j+2}$
$i+4$	correction	$A_{i+2}^{j-2} = 1,\ W_{i+3}^{j-2} = \bar{a}$
$i+5$	correction	$A_{i+3}^{j-2} = 0,\ W_{i+4}^{j-2} = \bar{a}$
$i+6$	correction	$W_{i+5}^{j-2} = \bar{a}$

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	-----------------------------a--	------------------------$\bar{a}$-------
02:	-------------------------------1	--------------------------------
03:	-------------------------------0	-------------------------------$\bar{a}$
04:	--------------------------------	-------------------------------$\bar{a}$
05:	--------------------------------	-------------------------------$\bar{a}$
06:	--------------------------------	--------------------------------

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
perturbation mask	1010000000100000	
differences on W^j	1010000000100000	0000000010110110
differences on W^{j+5}	0101000000010000	0000000001011011
differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------$\bar{a}$-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------$\bar{b}$------$\bar{a}$
04:	-------------------------------0	-------------------------------$\bar{a}$
05:	-------------------------------0	-------------------------------$\bar{a}$
06:	--------------------------------	-------------------------------$\bar{b}$
07:	--------------------------------	-------------------------------$\bar{b}$
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------$\bar{c}$-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------$\bar{c}$
15:	--------------------------------	-------------------------------$\bar{c}$

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
perturbation mask	1010000000100000	
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differences on W^{j+5}	0101000000010000	0000000001011011
differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------ā-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------b̄------ā
04:	-------------------------------0	-------------------------------ā
05:	-------------------------------0	-------------------------------ā
06:	--------------------------------	-------------------------------b̄
07:	--------------------------------	-------------------------------b̄
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------c̄-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------c̄
15:	--------------------------------	-------------------------------c̄

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
perturbation mask	1010000000100000	
differences on W^j	1010000000100000	0000000010110110
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differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------$\bar{a}$-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------$\bar{b}$------$\bar{a}$
04:	-------------------------------0	-------------------------------$\bar{a}$
05:	-------------------------------0	-------------------------------$\bar{a}$
06:	--------------------------------	-------------------------------$\bar{b}$
07:	--------------------------------	-------------------------------$\bar{b}$
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------$\bar{c}$-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------$\bar{c}$
15:	--------------------------------	-------------------------------$\bar{c}$

Building auxiliary differential paths

	W_0 to W_{15}	W_{16} to W_{31}
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differences on W^j	1010000000100000	0000000010110110
differences on W^{j+5}	0101000000010000	0000000001011011
differences on W^{j-2}	0001111100000011	0000000000001110

i	A_i	W_i
-1:	---------------------------d----	
00:	---------------------------d----	-----------------------------a--
01:	---------------------------e-a--	------------------------$\bar{a}$-------
02:	---------------------------e---1	-----------------------------b--
03:	-----------------------------b-0	------------------------$\bar{b}$------$\bar{a}$
04:	-------------------------------0	-------------------------------$\bar{a}$
05:	-------------------------------0	-------------------------------$\bar{a}$
06:	--------------------------------	-------------------------------$\bar{b}$
07:	--------------------------------	-------------------------------$\bar{b}$
08:	--------------------------------	--------------------------------
09:	---------------------------f----	--------------------------------
10:	---------------------------f----	-----------------------------c--
11:	-----------------------------c--	------------------------$\bar{c}$-------
12:	-------------------------------0	--------------------------------
13:	-------------------------------0	--------------------------------
14:	--------------------------------	-------------------------------$\bar{c}$
15:	--------------------------------	-------------------------------$\bar{c}$

Placing auxiliary differential paths

i	A_i	W_i
-4:	00101001010011011100100101000111	
-3:	00000111100001000110010101100010	
-2:	11011000010000101001111101011111	
-1:	01011011110111101101101111010001	
00:	01000010101101110111101110011011	1uu11101100111110110--0111111011
01:	n1n010111001011001001-0100100110	nuu101-10001011--111111101u1n0n1
02:	1nu11--01111101111101101111111u1	--n11-----0-10-1111000110n0111uu
03:	nnu00-----0-00-0110000110111110n	x-nn-1--1--01010001001--1u111001
04:	u010u11-0--00010010110-1010un0u1	uu-u0-------11-0--1011001n1n10nu
05:	1001u00-0--000000000001u00011010	nn-u0------11010111--1--11n100u1
06:	011unnnnnnnnnnnnnnn1---110n001uu	00n-------1-1--1--00111100011001
07:	u110-01000000u010110nu111uu1010n	1nu001------1--1-100-1-10-un-0n-
08:	1111010111111---011unu110-0--nu1	-un0----------11---------u0111nu
09:	-0010---1--1--01-0u-10nnnnu01010	--u0-------------1--1001-u1--100
10:	--------1--1--0--01-101nu1111u10	xxu00-----0--1--1--0--1--u----n-
11:	0---------0--1--1--0n-100nn0u1n0	-xn--1--0--0--1--0---11-0010--x-
12:	0---0-------0--0--0--01-010n1-nn	x------------------------------u
13:	00----------0--0--0--00100n0n-00	--10------------------0--1n1----
14:	-0--0-----------------10001u0un-	---1--------1--0--0--1--000---xn
15:	n-----------------------unnn1101	-x-10-------1--0--0--1--0u-n--u-
16:	--1---------------------1--nu001	-n0---------------------1u0-----
17:	n-0-----------------------111-0n	xxn-----------------1---1u-x--n-
18:	-11-------------------------101-	x-u1--------------------0----0--
19:	------------------------------u-	x----------------------11n------
20:	--------------------------------	--x----------------------------x
	...	...

Discussion on the implementation

- how to implement it ?
- we can use boomerang attacks with neutral bits or message modifications if we carefully check that the auxiliary paths remain valid.
- message modifications are costly and the 2^{63} attack is not yet published.
- works well with neutral bits (but their range is too small).

If you are interested in the details, see our paper !

Using auxiliary differential paths

- find a conformant message pair (with some auxiliary differential paths) and multiply it thanks to the neutral bits (check that a lot of the auxiliaries remain valid).
- when a message pair is conformant up to step 28, trigger the auxiliary paths and get new message pairs conformant up to step 28 for free.

M_0	11111101100111111111011111111011	0xfd9ff7fb
M_1	01110101000001010011111101110001	0x75053f71
M_2	00011100000111010111001100011111	0x1c1d731f
M_3	00000111001110000000001001111001	0x07380279
M_4	11110101101011101000100000101001	0xf5ae8829
M_5	00110101111110101100101101010011	0x35facb53
M_6	00010000011111001010101100011001	0x107cab19
M_7	10100110111111100110001101101001	0xa6fe6369
M_8	01001000001100111010100101011101	0x4833a95d
M_9	01100000000110110110100111101100	0x601b69ec
M_{10}	10100011010010100100111001100100	0xa34a4e64
M_{11}	01011100100111101011111100100111	0x5c9ebf27
M_{12}	10111011010000110101001001110111	0xbb435277
M_{13}	10100101011101110100110011010100	0xa5774cd4
M_{14}	11111110011110111011010000000000	0xfe7bb400
M_{15}	10110101001110111010110101101011	0xb53bad6b

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Yet another way of using freedom degrees ...

- boomerang attack for hash functions is nothing more than another way of cleverly using the freedom degrees from the message.
- message modifications, neutral bits, auxiliary differentials are closely related.
- they all have pros and cons:

	message modifications	neutral bits	auxiliary paths
speed cost	big	medium	small
freedom degrees cost	medium	small	big
range	medium	small	long

... but freedom degrees are not unlimited !

- we can not use all those techniques independently !
- twofold waste of freedom degrees: or we use a lot of freedom degrees for a small gain, or some freedom degrees are left unused.
- it would be great to find a way to use exactly what we need from all those techniques.
- not trivial since we need to settle the long range characteristics first, which imposes a lot (too much ?) of constraints.
- maybe a generalization of those techniques may achieve this ?

That's all folks !

Thank you !