Cryptanalysis of the ESSENCE Hash Function

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Outline

ESSENCE

ESSENCE [Jason W. Martin]

- First round candidate of the NIST SHA-3 competition
  - 64 submissions (October 2008)
  - 51 first round candidates
  - 14 second round candidates (July 2009)

- Based on feedback shift registers
  - over 32-bit words for ESSENCE-256/224
  - over 64-bit words for ESSENCE-512/384
- Message block: 8 words
- Chaining value: 8 words
- Merkle-Damgård tree
- Davies-Meyer construction for the compression function

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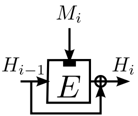
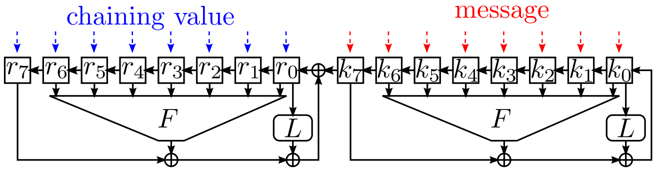
Compression Function

- Davies-Meyer construction



- Block Cipher



$32\times$ clocked

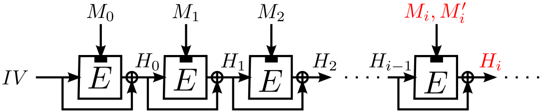
  - F: bitwise non-linear function
  - L: linear function on the whole word
  - 32 reversible steps

Attack on ESSENCE

Principle

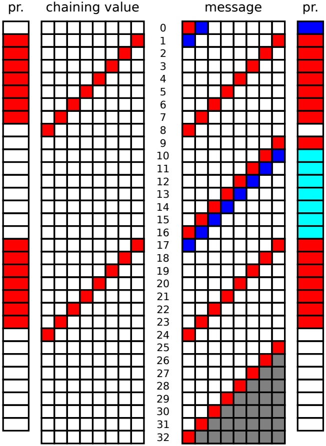
- Collision attack
  - Find $\mathcal{M} \neq \mathcal{M}'$ so that $\mathcal{H}(\mathcal{M}) = \mathcal{H}(\mathcal{M}')$
  - Complexity of generic attack: $2^{\ell_h/2}$
    where $\ell_h = |\mathcal{H}(\mathcal{M})|$

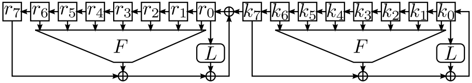
- For a chaining value H_{i-1} find two messages M_i, M_i' that collide to the same value H_i



- Using a differential path

Differential Path





Differences:

- □ no difference
- red: α
- blue: $\beta = L(\alpha)$
- grey: unknown

Probabilities:

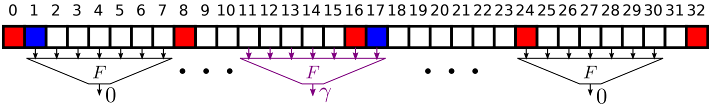
- red: $2^{-|\alpha|}$
- blue: $2^{-|\beta|}$
- cyan: $2^{-|\alpha \vee \beta|}$
- □ 1

Condition:

$$\alpha \vee \beta \vee L(\beta) = \alpha \vee \beta$$

Exact Complexities

- Probabilities based on Hamming weight (HW) are not accurate enough:
  - e.g. a 1 bit difference has probability $2^{-8.4}$ to be canceled in the 7 steps of F, and not 2^{-7} as we would guess from the HW
- For accurate estimates consider the whole path bitwise
  - Possible differences: $(\alpha_i, \beta_i, \gamma_i)$ with $0 \leq i \leq 32/64$ and $\beta = L(\alpha)$ and $\gamma = L(\beta)$
  - Have to test 2^{30} values for each each $(\alpha_i, \beta_i, \gamma_i)$



Probability of Complete Path - Bitwise

- Bitwise probability, independent of α

$(\alpha_i, \beta_i, \gamma_i)$	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)
probability	1	0	$2^{-9.5}$	$2^{-9.1}$
$(\alpha_i, \beta_i, \gamma_i)$	(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)
probability	$2^{-24.4}$	0	2^{-23}	2^{-26}

- Gives two conditions for α:
  - $\neg\alpha \wedge \neg\beta \wedge \gamma = 0$
  - $\alpha \wedge \neg\beta \wedge \gamma = 0$

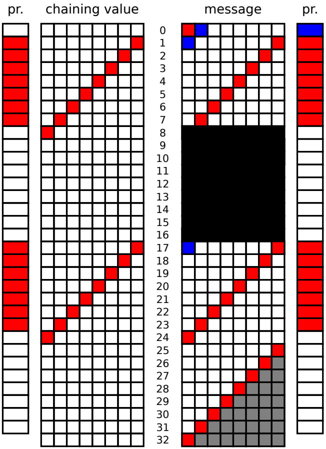
Complexity of Complete Path

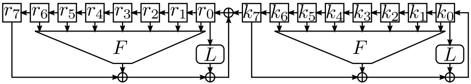
- Complexity for the α's used in our attack:

	differential path left	differential path right	generic method
ESSENCE-256	$2^{67.4}$	$2^{240.6}$	2^{128}
ESSENCE-512	$2^{134.7}$	$2^{478.9}$	2^{256}

- About $2^{15.4}$ pairs follow the whole path for ESSENCE-256 ($2^{37.1}$ for ESSENCE-512)

Idea: Computing the Middle Part





Differences:

- □ no difference
- ■ (red) α
- ■ (blue) $\beta = L(\alpha)$
- ■ (grey) unknown
- ■ (black) precomputed

Probabilities:

- (red) $2^{-|\alpha|}$
- (blue) $2^{-|\beta|}$
- (cyan) $2^{-|\alpha \vee \beta|}$
- (white) 1

Conditions:

$$\neg\alpha \wedge \neg\beta \wedge \gamma = 0$$
$$\alpha \wedge \neg\beta \wedge \gamma = 0$$

Strategy of the Attack

- Compute many pairs that fulfill the middle part (step 8-17)
- Search among those one message pair that follows the rest of the path (step 0-8 and step 17-32)
- Try different chaining values (random starting messages) with our message pair to find a collision

Computing the Middle Part

8	$x_0 \oplus \alpha$	x_1	x_2	x_3	x_4	x_5	x_6	x_7
9	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$
10	x_2	x_3	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$
11	x_3	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}
12	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}
13	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}
14	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}
15	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
16	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
17	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	$x_{16} \oplus \alpha$

- Let ℓ be the word size (32 or 64), $\beta = L(\alpha)$, $\gamma = L(\beta)$, $s = |\alpha \vee \beta|$ and $\mathcal{S} = \{i : \alpha_i \vee \beta_i = 1\}$

Computing the Middle Part - Bit Level

- For all bit-difference $(\alpha_i, \beta_i, \gamma_i)$, $0 \leq i < 32/64$:
  - Store bit-tuples $(x_1, \ldots, x_{15})_i$ passing F in the middle part:

    $$e.g. : F(x_2, x_3, x_4, x_5, x_6, x_7, x_8\ )_i = F(x_2, x_3, x_4, x_5, x_6, x_7, x_8 \oplus \alpha)_i$$

  - Better: Store only those tuples which have a possibility to follow the rest of the path
- Number of adequate tuples depending on the bit-differences:

$(\alpha_i, \beta_i, \gamma_i)$	$(0,0,1)$	$(0,1,0)$	$(0,1,1)$	$(1,0,0)$	$(1,0,1)$	$(1,1,0)$	$(1,1,1)$
	0	96	128	96	120	96	176
better	0	96	128	2	0	4	2

- Number of possibilities to choose $(x_1, \ldots, x_{15})_i$, $i \in \mathcal{S}$:

$$N_\alpha = 2^{|\alpha \wedge \neg\beta \wedge \neg\gamma|} \times 4^{|\alpha \wedge \beta \wedge \neg\gamma|} \times 96^{|\neg\alpha \wedge \beta \wedge \neg\gamma|} \times 2^{|\alpha \wedge \beta \wedge \gamma|} \times 128^{|\neg\alpha \wedge \beta \wedge \gamma|}$$

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Computing the Middle Part - Fix s Bits

$$L(\overbrace{x_7}^{s \text{ bits fixed}}) = x_0 \oplus \overbrace{x_8 \oplus F(x_1, x_2, x_3, x_4, x_5, x_6, x_7)}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_8}^{s \text{ bits fixed}}) = \overbrace{x_1 \oplus x_9 \oplus F(x_2, x_3, x_4, x_5, x_6, x_7, x_8 \oplus \alpha)}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_9}^{s \text{ bits fixed}}) = \overbrace{x_2 \oplus x_{10} \oplus F(x_3, x_4, x_5, x_6, x_7, x_8 \oplus \alpha, x_9 \oplus \beta) \oplus \gamma}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_{10}}^{s \text{ bits fixed}}) = \overbrace{x_3 \oplus x_{11} \oplus F(x_4, x_5, x_6, x_7, x_8 \oplus \alpha, x_9 \oplus \beta, x_{10})}^{s \text{ bits fixed}}$$

$$\ldots$$

$$L(\overbrace{x_{14}}^{s \text{ bits fixed}}) = \overbrace{x_7 \oplus x_{15} \oplus F(x_8 \oplus \alpha, x_9 \oplus \beta, x_{10}, x_{11}, x_{12}, x_{13}, x_{14})}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_{15}}^{s \text{ bits fixed}}) = x_{16} \oplus \overbrace{x_8 \oplus F(x_9 \oplus \beta, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15})}^{s \text{ bits fixed}}$$

Computing the Middle Part - Linear Systems

- We have 7 linear systems depending on α, $8 \leq j \leq 14$

$$L(x_j) = R_j$$

- x_j and R_j have together
  - 2ℓ bits (ℓ is the word length)
  - $2s$ bit fixed
- L gives ℓ equations
- Probability of a solution $2^{-(2s-\ell)}$ if the system has full rank

Computing the Middle Part - Solving the Systems

- The position of the fixed bits is given by $\mathcal{S}$
- Using Gauss elimination we find $2s - \ell$ equations which must be satisfied to have a solution
- Order the $7(2s - \ell)$ equations depending on the variables they contain, so that changing the variables in the later equations has no influence on the results of the first ones

Computing the Middle Part - Finishing

- After solving the linear systems we have
  - In x_j,R_j all bits fixed, $8 \leq j \leq 14$
  - In $x_1, \ldots, x_7, x_{15}$ we have s bits fixed
  - In x_0, x_{16} no bit fixed
- Selecting the $\ell - s$ free bits of x_7 allows us to determine all the other free bits
  $\Rightarrow$ For each solution of the linear systems we have $2^{\ell - s}$ solutions for the middle part for free
- In average, we find a solution for $x_0, \ldots, x_{16}$ in less than one call to the compression function

Final Complexity

- To find the optimal α
  - ESSENCE-256: Test all possible α
  - ESSENCE-512: Test all α's with HW ≤ 8 (limitation on the left side)

	differential path		generic method
	left	right	
ESSENCE-256	$2^{67.4}$	$2^{62.2}$	2^{128}
ESSENCE-512	$2^{134.7}$	$2^{116.1}$	2^{256}

Semi-Free-Start Collision on 29 rounds

	Initial values for r									Initial values for k								
	B0741769	BA2BA1A1	349A4DC8	54204D82	292006B1	80096194	D23020E1	9098A7EA		4CD35806	4759FB6D	3ED267E5	17641536	BE1F35ED	688B0C3C	DF126549	5FAE0827	
round	differences								round	differences								round
0	0	0	0	0	0	0	0	0	0	80102040	537874EB	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	80102040	1	537874EB	0	0	0	0	0	0	80102040	1
2	0	0	0	0	0	0	80102040	0	2	0	0	0	0	0	0	80102040	0	2
3	0	0	0	0	0	80102040	0	0	3	0	0	0	0	0	80102040	0	0	3
4	0	0	0	0	80102040	0	0	0	4	0	0	0	0	80102040	0	0	0	4
5	0	0	0	80102040	0	0	0	0	5	0	0	0	80102040	0	0	0	0	5
6	0	0	80102040	0	0	0	0	0	6	0	0	80102040	0	0	0	0	0	6
7	0	80102040	0	0	0	0	0	0	7	0	80102040	0	0	0	0	0	0	7
8	80102040	0	0	0	0	0	0	0	8	80102040	0	0	0	0	0	0	0	8
9	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0	80102040	9
10	0	0	0	0	0	0	0	0	10	0	0	0	0	0	0	80102040	537874EB	10
11	0	0	0	0	0	0	0	0	11	0	0	0	0	0	80102040	537874EB	0	11
12	0	0	0	0	0	0	0	0	12	0	0	0	0	80102040	537874EB	0	0	12
13	0	0	0	0	0	0	0	0	13	0	0	0	80102040	537874EB	0	0	0	13
14	0	0	0	0	0	0	0	0	14	0	0	80102040	537874EB	0	0	0	0	14
15	0	0	0	0	0	0	0	0	15	0	80102040	537874EB	0	0	0	0	0	15
16	0	0	0	0	0	0	0	0	16	80102040	537874EB	0	0	0	0	0	0	16
17	0	0	0	0	0	0	0	80102040	17	537874EB	0	0	0	0	0	0	80102040	17
18	0	0	0	0	0	0	80102040	0	18	0	0	0	0	0	0	80102040	0	18
19	0	0	0	0	0	80102040	0	0	19	0	0	0	0	0	80102040	0	0	19
20	0	0	0	0	80102040	0	0	0	20	0	0	0	0	80102040	0	0	0	20
21	0	0	0	80102040	0	0	0	0	21	0	0	0	80102040	0	0	0	0	21
22	0	0	80102040	0	0	0	0	0	22	0	0	80102040	0	0	0	0	80000040	22
23	0	80102040	0	0	0	0	0	0	23	0	80102040	0	0	0	0	80000040	38C32419	23
24	80102040	0	0	0	0	0	0	0	24	80102040	0	0	0	0	80000040	38C32419	3B50EAEF	24
25	0	0	0	0	0	0	0	0	25	0	0	0	0	80000040	38C32419	3B50EAEF	E9F738F8	25
26	0	0	0	0	0	0	0	0	26	0	0	0	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	26
27	0	0	0	0	0	0	0	0	27	0	0	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	27
28	0	0	0	0	0	0	0	0	28	0	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	28
29	0	0	0	0	0	0	0	0	29	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	29
30	0	0	0	0	0	0	0	80000040	30	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	A7EF91F9	30
31	0	0	0	0	0	0	80000040	102040	31	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	A7EF91F9	21E1C70	31
32	0	0	0	0	0	80000040	102040	3336DACE	32	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	A7EF91F9	21E1C70	1B715D5F	32

Conclusion

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- **Complexity:**
  - ESSENCE-*256*: $2^{67.4}$
  - ESSENCE-*512*: $2^{134.7}$

- **Why does the attack work?**
  - Message processing is independent of chaining value
  - Precompute low probability part
  - Efficient solving of linear system
  - Very accurate probability estimation by considering the bit path
  - Reduced cost by considering the whole path

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